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NON-EXISTENCE OF PHOTON SPIN IN CONVENTIONAL THEORY

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ABSTRACT: Maxwell's equations have been used as a guideline in conventional quantum electrodynamics. Also the related Poynting vector has thereby formed the basis for the quantized field momentum. In this letter the conventional theory is applied to the model of a unidirectionally propagating photon, as described in a cylindrical frame of reference. Due to the vanishing electric field divergence of Maxwell's equations in the vacuum, the photon field is then found to be purely transverse, having a Poynting vector and energy flow in the axial direction of propagation only, and leading to the unacceptable result of a nonexisting angular momentum (spin). One possible way of avoiding this shortcoming is provided by a revised electromagnetic theory which includes the additional degree of freedom of a nonzero electric field divergence in the vacuum state.

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1. INTRODUCTION

In a conventional quantum electrodynamical (QED) approach to the physics of elementary particles and light, Maxwell's equations with a vanishing electric field divergence in the vacuum have been used as a guideline and basis as shown by Schiff [9] among others. This is reconcilable with the fact that the quantized equations become equivalent to Maxwell's equations for the expectation values of the electromagnetic field quantities, as shown by Heitler [3]. Conventional theory based on Maxwell's equations and quantum mechanics has in this way been very successful in its applications to numerous problems in physics, and has sometimes manifested itself in an extremely good agreement with experiments.

Nevertheless there are areas within which these joint theories do not provide fully adequate descriptions of physical reality. As already stated by Feynman [2], there are difficulties associated with the *ideas* of Maxwell's theory which are not solved by

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and not directly associated with quantum mechanics. Thus the classical theory of electromagnetism is in its conventional form an unsatisfactory theory of its own. In this paper the shortcomings of the conventional theory will be explicitly demonstrated by one straightforward and outstanding example, namely that of the required angular momentum (spin) of an individual photon in its capacity of a boson particle.

2. MAXWELL'S EQUATIONS IN THE VACUUM STATE

Here we first turn to the classical electromagnetic theory of Maxwell's equations in the vacuum, as given by

$$\operatorname{curl}(\mathbf{B}/\mu_{\mathbf{0}}, \mathbf{E}) = \frac{\partial}{\partial \mathbf{t}}(\varepsilon_{\mathbf{0}}\mathbf{E}, -\mathbf{B}) \tag{1}$$

in SI units for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, where μ_0 and ε_0 stand for the permeability and permittivity. Divergence operations on these equations result in

$$\operatorname{div}(\mathbf{E}, \mathbf{B}) = \mathbf{0} \tag{2}$$

if the same fields are assumed to vanish at some time in their past. Combination of expressions (1) yields

$$\left(\operatorname{curl}\,\operatorname{curl} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(\mathbf{E}, \mathbf{B}) = 0 \tag{3}$$

where $c = (1/\mu_0\varepsilon_0)^{1/2}$ is the velocity of light. In this connection the density of the angular momentum becomes

$$\mathbf{s} = \mathbf{r} \times \mathbf{S}\,, \quad \mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_{\mathbf{0}}\,, \tag{4}$$

according to Morse and Feshbach [8], where $\mathbf{r}$ is the radius vector directed from the origin of the frame being used, and $\mathbf{S}$ is the Poynting vector. The latter is generally conceived to represent the magnitude and direction of the energy flow in space. There is full symmetry between the fields $\mathbf{E}$ and $\mathbf{B}$ in equations (2) and (3).

3. A CONVENTIONAL MODEL OF THE INDIVIDUAL PHOTON

When working out a model of the individual photon as a propagating boson, a wave or a wave packet with preserved geometrical shape and undamped motion in a defined direction of space has to be taken as a starting-point. This leads to the concepts of plane and cylindrical waves. The orientation of the frame of reference is then for the sake of simplicity chosen with the z axis in the direction of propagation. We thus assume every field quantity Q to vary as

$$Q = \overline{Q}e^{i\theta} \equiv qe^{i(-\omega t+kz)}\,, \tag{5}$$

where ω is the frequency, k the wave number in the direction of propagation, and the phase and group velocities are equal to $c = \omega/k$. Three alternatives are examined here as follows.

(i) When q =const., a rectangular frame (x, y, z) is introduced, and there is a plane wave. According to equations (2) the components E_z and B_z disappear, the wave becomes transverse, the Poynting vector (4) has a component in the direction of propagation only, and the angular momentum density s_z as well as the total spin vanish, as has also been shown earlier by Heitler [3]. A plane electromagnetic wave can therefore not represent an individual photon with spin. In addition, its energy is spread over an infinite volume of space. The same conclusion applies to a spherical wave at large distances from its source according to the solutions given by Morse and Feshbach [8].

(ii) When $q = f(r)$ in a cylindrical frame (r, φ, z), there is a purely axisymmetric cylindrical wave. The solutions of equations (3) for the electric field components are

$$\overline{E}_r = c_{r1}r + c_{r2}/r\,, \quad \overline{E}_\varphi = c_{\varphi 1}r + c_{\varphi 2}/r\,, \quad \overline{E}_z = c_{z1}\ln r + c_{z2}\,, \tag{6}$$

where c_{r1} to c_{z2} are arbitrary constants. Analogous relations hold for the magnetic field. The infinite limits of these solutions at large and small r introduce problems when photon models of limited transverse extensions in space are being aimed at. This was already realized by Thomson [10]. However, an even more serious shortcoming arises from the requirements (2) of a vanishing field divergence. Since the second order equations (3) have been obtained from a combination of expressions (1), the solutions (6) have to be checked with respect to the first order equations (2). For the electric field the identity

$$2c_{r1} + ik(c_{z1}\ln r + c_{z2}) \equiv 0 \tag{7}$$

has thus to be satisfied for all (r, k). This implies that E_z and c_{r1} have to vanish, and only c_{r2}, $c_{\varphi 1}$, and $c_{\varphi 2}$ become nonzero. Analogous results apply to the magnetic field. Here again, there is a transverse electromagnetic wave with a Poynting vector in the z direction only, and a vanishing spin.

(iii) When $q = f(r)\exp(im\varphi)$ with $m \neq 0$ in a cylindrical frame, there is a screw-shaped(twisted) cylindrical wave. As described by Battersby [1], twisted light has recently attracted considerable interest, due to its possible applications to communication and microbiology. Equations (3) now result in the system

$$\left(D - \frac{m^2}{r^2}\right)\overline{E}_z = 0\,, \quad D = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\,, \tag{8}$$

$$\left(D - \frac{1+m^2}{r^2}\right)\left(\overline{E}_r, i\overline{E}_\varphi\right) - \frac{2m}{r^2}\left(i\overline{E}_\varphi, \overline{E}_r\right) = 0\,. \tag{9}$$

The solutions become

$$\overline{E}_z = [c_{1z}r^m + c_{2z}r^{-m}]e^{im\varphi}\,, \tag{10}$$

and

$$\overline{E}_r = \left[c_{1r}\mathrm{r}^{1\pm m} + c_{2r}\mathrm{r}^{-(1\pm m)}\right]e^{im\varphi} = \pm i\overline{E}_\varphi\,, \quad 1 \pm m \neq 0\,, \tag{11}$$

$$\overline{E}_r = \left[c_{1ro} + c_{2ro}\ln r\right]e^{im\varphi} = \pm i\overline{E}_\varphi\,, \quad 1 \pm m = 0\,, \tag{12}$$

where c_{1z} to c_{2ro} are arbitrary constants. Condition (2) yields

$$\frac{1}{r}\frac{\partial}{\partial r}(r\overline{E}_r) + \frac{m}{r}\left(i\overline{E}_\varphi\right) + ik\overline{E}_z \equiv 0 \tag{13}$$

to be satisfied for all (r, m, k). Insertion of the solutions (10)–(12) into condition (13) makes $\overline{E}_z$ vanish, as well as c_{1r} when $1 \pm m \neq 0$, and c_{2ro} when $1 \pm m = 0$. As in alternative (ii), there is a transverse wave with problems concerning the desired limited extensions in space, a Poynting vector in the z direction only, and no spin.

To sum up, there is a shortcoming of classical electromagnetic theory in providing a guideline for a model of an individual spatially limited and unidirectionally propagating photon. Thus, it is seen that such a theory yields purely transverse waves, a Poynting vector and energy flow in the direction of propagation only, and a non-existing angular momentum (spin). It applies to plane waves, spherical waves at large distances from their source, to purely axisymmetric cylindrical waves, and to cylindrical screw-shaped (twisted) waves. This shortcoming can be interpreted as the consequence of a vanishing electric field divergence.

The limited features deduced from Maxwellian theory are also transferred to a fully quantized (QED) analysis. According to Schiff [9] and Heitler [3] the Poynting vector namely forms as well the basis for the quantized field momentum. The electromagnetic field strengths are then expressed in terms of corresponding quantum mechanical wave expansions.

4. THE POSSIBILITY OF A REVISED ELECTROMAGNETIC THEORY

There is at least one possible way of overcoming this dilemma, namely by removing the symmetry between the electric and magnetic fields and adopting the degree of freedom of a nonzero electric field divergence in the vacuum state. Such an approach has been elaborated by Lehnert [4], [5], [6], [7] a detailed description of which is outside the frame of this short letter. Its starting-point is a generalized four-dimensional and inhomogenous Proca-type form

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) A_\mu = \mu_0 J_\mu \quad \mu = 1, 2, 3, 4\,, \tag{14}$$

where $A_\mu = (\mathbf{A}, i\phi/c)$ with $\mathbf{A}$ and ϕ standing for the magnetic vector potential and the electrostatic potential, and

$$J_\mu = (\mathbf{j}, ic\rho) \tag{15}$$

as an additional four-current density due to the possible sources of the field which consist of a three-space current density $\mathbf{j}$ and an electric charge density ρ. When J_μ vanishes, relation (14) reduces to the d'Alembert equations which lead back to equations (1). The right-hand member of equations (14) is now given a new interpretation in the vacuum state. As being supported by the positron-electron pair formation out of the vacuum, and by the observed vacuum fluctuations of the zero-point field, a nonzero electric space-charge density ρ is able to arise in the vacuum, as well as an associated electric field divergence. A preserved Lorentz invariance of equation (14) and of the four-current density (15) then leads to the three-space current density

$$\mathbf{j} = \rho\mathbf{C}\,, \quad \rho = \varepsilon_0\,(\,\mathrm{div}\,\mathbf{E})\,, \quad \mathbf{C}^2 = c^2\,. \tag{16}$$

Here $\mathbf{C}$ is a velocity vector having a modulus equal to c. In three-space the revised and extended field equations for the vacuum thus include a current density $\mathbf{j}$ in addition to the displacement current, and an associated nonzero charge density ρ.

Application of the revised field equations (14)–(16) to the cylindrical waves of cases (ii) and (iii) then leads to photon models having helical forms of the electromagnetic field and the Poynting vector, limited spatial extensions, no net electric charge and magnetic moment, but with a nonzero angular momentum, as shown by Lehnert [6], Lehnert [7].

5. CONCLUSIONS

As long as the Poynting vector concept remains valid both in electromagnetic field theory and in quantum electrodynamics, a nonzero photon spin cannot result from conventional theory. This shortcoming is due to the vanishing electric field divergence in the latter theory, and it can be avoided in a revised electromagnetic theory which is based on a nonzero electric field divergence in the vacuum state.

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