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FINITE-TIME STOCHASTIC SYNCHRONIZATION FOR A CLASS OF BAM NEURAL NETWORKS WITH UNCERTAIN PARAMETERS

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ABSTRACT. This paper is concerned with the finite-time stochastic synchronization for a class of bidirectional associative memory (BAM) neural networks with uncertain parameters. With the adaptive control method, sufficient conditions for finite-time stochastic synchronization and parameters identification are derived based on finite-time stability theory of stochastic differential equations. We also provide a numerical example to support the effectiveness of the proposed method.
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1. INTRODUCTION

Since bidirectional association memory (BAM) neural networks were proposed by Kosko [1] in 1987, many researches have done extensive works on this subject due to its successful application in pattern recognition and artificial intelligence. The stability for BAM neural networks has gained much attention, and a large amounts of related results have been presented (see [2-4] and references therein).

It is well known that studies on neural dynamical systems not only involve in the discussion of stability, but also involve many other dynamical behaviors such as oscillation, bifurcation, chaos and so on. Recently, chaos synchronization [5,6] for neural networks have been studied and a wide variety of synchronization approaches have been carried out such as impulsive control method [7,8], adaptive control method [9,10], feedback control method [11] and so on.

However, a lot of existing papers studying the synchronization of neural networks focus on the cases that system parameters are exactly known in advance. Obviously, this assumption is not appropriate in many practical situations. For this reason,

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papers [12-15] investigate the parameter identification problem based on adaptive control methods. Besides uncertain parameter, it is usually inevitable affected by the stochastic noises in real nervous system. Especially, these uncertain factors can affect the dynamical behaviors of systems and make the systems unstable and oscillatory. Some interesting results about synchronization for neural network with uncertain parameters and stochastic disturbance have been obtained [16-18].

On the other hand, the time of synchronization is also a research topic [19,20]. Because synchronizability is affected by many topological features of networks, so much work is devoted to design the optional graph topology [21]. Although we can get better convergent rate by this way, convergence occurs asymptotically and convergent time is infinite. In practical situations, it is often required that synchronization be reached in finite time. Finite-time synchronization means that the states of two or multiple dynamical systems achieve synchronization in finite time. For instance, in secure communication, the range of time during which the oscillators are not synchronized correspond to the range of time during which the encoded message cannot be recovered or sent [22]. It is obvious that finite-time synchronization is more important and practical than the asymptotical synchronization. Therefor, it is significant to consider finite-time synchronization of neural networks. To the best of our knowledge, however, few results have been presented about finite-time synchronization for a class of BAM neural networks with uncertain parameters and stochastic noise perturbation.

Motivated by the above discussion. In this paper, our aim is to design adaptive controller and updated laws to realize the finite-time stochastic synchronization for a class of BAM neural networks with uncertain parameters. This paper is organized as follows: In Section 2, model description and preliminaries are presented. In Section 3, an adaptive controller and update laws for a class of BAM neural networks with uncertain parameters and noise perturbation are proposed, basing on a finite-time stability theory for stochastic differential equations. In Section 4, an numerical example is provided to illustrate the validity of the theoretical results. Finally, conclusion is summarized in Section 5.

2. PRELIMINARIES

Throughout this paper,unless otherwise specified, we use the following notations. Let $\mathrm{R^n}$ denotes the n-dimensional Euclidean space with the norms $\|x\|_1 = \sum_{i=1}^{n} |x_i|$, $\|x\|_2 = (\sum_{i=1}^{n} x_i^2)^{1/2}$ and $\|x\|_\infty = \max_{1\le i\le n}\{|x_i|\}$, where $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in \mathrm{R}^n$. Let $\mathrm{sign}(x) = (\mathrm{sign}(x_1), \mathrm{sign}(x_2), \cdots, \mathrm{sign}(x_n))^{\mathrm{T}}$, where $\mathrm{sign}(\cdot)$ is the sign function. The notation $X \ge Y$ $(X > Y)$, where X and Y are symmetric matrices, means that $X-Y$ is a symmetric positive semi-definite (positive definite) matrix. For a given square matrix A, A^{T} denotes its transpose and $\mathrm{tr}\{A\}$ denotes its trace. $\mathbb{E}\{\cdot\}$ represents the expectation operator.

Let$(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. A $d\times m$-valued measurable function $G = G(s,\omega)$ defined on $[t_0, t]\times\Omega$ is said to be nonanticipating if $G = G(s,\cdot)$ is $\mathcal{F}_s$ measurable for all $s\in[t_0, t]$.

In this paper,we consider follwing BAM neural networks with stochastic perturbation as drive systems:

$$\begin{cases} \mathrm{d}x(t) = (-C(x(t)) + W^{\mathrm{T}}F(y(t)) + \Delta g_1(t, x(t)) + I)\mathrm{d}t + \varphi_1(x(t))\mathrm{d}\omega(t), \\ \mathrm{d}y(t) = (-D(y(t)) + HG(x(t)) + \Delta h_1(t, y(t)) + J)\mathrm{d}t + \psi_1(y(t))\mathrm{d}\nu(t), \end{cases} \tag{2.1}$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_m(t))^{\mathrm{T}}$ and $y(t) = (y_1(t), y_2(t), \cdots, y_n(t))^{\mathrm{T}}$ correspond to the state vectors of drive system; $C(x(t)) = (c_i(x_i(t)))_{m\times 1}$ and $D(y(t)) = (d_j(y_j(t))_{n\times 1}$ are appropriately behaved functions; $W^{\mathrm{T}} = (w_{ji})_{m\times n}$ and $H = (h_{ij})_{n\times m}$ are connection weight matrices; $F(y(t)) = (f_j(y_j(t)))_{n\times 1}$ and $G(x(t)) = (g_i(x_i(t)))_{m\times 1}$ are the nonlinear activation functions; $\Delta g_1(t, x(t)) : \mathrm{R}^+\times\mathrm{R}^m \to \mathrm{R}^m$ and $\Delta h_1(t, y(t)) : \mathrm{R}^+\times\mathrm{R}^n \to \mathrm{R}^n$ denote uncertain terms, respectively; $I = (I_i)_{m\times 1}$ and $J = (J_j)_{n\times 1}$ are external bias; $\varphi_1(x(t)) : \mathrm{R}^m \to \mathrm{R}^{m\times m}$ and $\psi_1(y(t)) : \mathrm{R}^n \to \mathrm{R}^{n\times n}$ denote the diffusion matrices and are nonanticipating; $\omega(t)$ and $\nu(t)$ are two independent standard m-dimensional and n-dimensional Wiener process defined on $(\Omega, \mathcal{F}, P)$.

The corresponding response systems are given as follows:

$$\begin{cases} \mathrm{d}u(t) = (-C(u(t)) + \hat{W}^{\mathrm{T}}(t)F(v(t)) + \Delta g_2(t, u(t)) \\ \qquad +I + p(t))\mathrm{d}t + \varphi_2(u(t))\mathrm{d}\omega(t), \\ \mathrm{d}v(t) = (-D(v(t)) + \hat{H}(t)G(u(t)) + \Delta h_2(t, v(t)) + J + q(t))\mathrm{d}t \\ \qquad +\psi_2(v(t))\mathrm{d}\nu(t), \end{cases} \tag{2.2}$$

where $u(t) = (u_1(t), u_2(t), \cdots, u_m(t))^{\mathrm{T}}$ and $v(t) = (v_1(t), v_2(t), \cdots, v_n(t))^{\mathrm{T}}$ are the state vectors of response system; $\hat{W}^{\mathrm{T}}(t) = (w_{ji}(t))_{m\times n}$, $\hat{H}(t) = (h_{ij}(t))_{n\times m}$ represent unknown connection weight matrices at time t; $\Delta g_2(x(t)) : \mathrm{R}^m \to \mathrm{R}^m$ and $\Delta h_2(y(t)) : \mathrm{R}^n \to \mathrm{R}^n$ denote uncertain terms, respectively; $I = (I_i)_{m\times 1}$ and $J = (J_j)_{n\times 1}$ are external bias; $p(t) = (p_1(t), p_2(t), \cdots, p_m(t))^{\mathrm{T}}$ and $q(t) = (q_1(t), q_2(t), \cdots, q_n(t))^{\mathrm{T}}$ are external control input vectors; $\varphi_2(u(t)) : \mathrm{R}^+\times\mathrm{R}^m \to \mathrm{R}^{m\times m}$ and $\psi_2(v(t)) : \mathrm{R}^+\times\mathrm{R}^n \to \mathrm{R}^{n\times n}$ denote the diffusion matrices and are nonanticipating;

Definition 2.1. *The drive systems (2.1) and the response systems (2.2) are said to be the finite-time stochastic synchronization if there exists a positive constant $T > t_0 \geq 0$ such that*

$$\lim_{t\to T^-} \mathbb{E}\{\|u(t) - x(t)\|_2\} = 0 \textit{ and } \lim_{t\to T^-} \mathbb{E}\{\|v(t) - y(t)\|_2\} = 0,$$

moveover,

$$\mathbb{E}\{\|u(t) - x(t)\|_2\} = 0 \textit{ and } \mathbb{E}\{\|v(t) - y(t)\|_2\} = 0, \quad t \geq T,$$

for any initial conditions $x(t_0)$,$y(t_0)$,$u(t_0)$ and $v(t_0)$.

Remark 2.2. Here $T = T(x(t_0), y(t_0), u(t_0), v(t_0), \omega)$ is a stochastic variable with initial values $x(t_0)$,$y(t_0)$,$u(t_0)$ and $v(t_0)$. Hence, the finite-time T is evaluate by $0 < \mathbb{E}(T) < +\infty$.

Define the synchronization error as $e_x(t) = u(t) - x(t)$, $e_y(t) = v(t) - y(t)$, then we have the following error systems:

$$\begin{cases} \mathrm{d}e_x(t) = (-C(e_x(t)) + \hat{W}^{\mathrm{T}}F(v(t)) - W^{\mathrm{T}}F(y(t)) + \Delta g_2(t, u(t)) - \Delta g_1(t, x(t)) \\ \qquad +p(t))\mathrm{d}t + (\varphi_2(u(t)) - \varphi_1(x(t)))\mathrm{d}\omega(t), \\ \mathrm{d}e_y(t) = (-D(e_y(t)) + \hat{H}G(u(t)) - HG(x(t)) + \Delta h_2(t, v(t)) - \Delta h_1(t, y(t)) \\ \qquad +q(t))\mathrm{d}t + (\psi_2(v(t)) - \psi_1(y(t)))\mathrm{d}\nu(t), \end{cases} \tag{2.3}$$

where $C(e_x(t)) = C(u(t)) - C(x(t))$, $D(e_y(t)) = D(v(t)) - D(y(t))$.

In this paper, we make the following assumptions.

(H_1) Behaved functions $c_i(\cdot)$ and $d_j(\cdot)$ are continuously differentiable and satisfy $\dot{c}_i(\cdot) \geq c_i > 0$, $\dot{d}_j(\cdot) \geq d_j > 0$ for $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$.

(H_2) There exist nonnegative constants $a_i \geq 0$, $b_i \geq 0$ for $i = 1, 2.$, such that $\|\Delta g_1(t, x(t))\|_\infty \leq a_1$, $\|\Delta g_2(t, u(t))\|_\infty \leq a_2$, $\|\Delta h_1(t, y(t))\|_\infty \leq b_1$ and $\|\Delta h_2(t, v(t))\|_\infty \leq b_2$ for any $x(t), u(t) \in \mathrm{R}^m$, $y(t), v(t) \in \mathrm{R}^n$ and $t \geq t_0$.

(H_3) There exist nonnegative constants $\delta_i \geq 0$ for $i = 1, 2.$, such that $\mathrm{tr}(\varphi_2(u(t)) - \varphi_1(x(t)))^{\mathrm{T}}(\varphi_2(u(t)) - \varphi_1(x(t))) \leq \delta_1$, $\mathrm{tr}(\psi_2(v(t)) - \psi_1(y(t)))^{\mathrm{T}}(\psi_2(v(t)) - \psi_1(y(t))) \leq \delta_2$ for any $x(t), u(t) \in \mathrm{R}^m$, $y(t), v(t) \in \mathrm{R}^n$ and $t \geq t_0$.

(H_4) The elements of matrices $W^{\mathrm{T}} = (w_{ji})_{m \times n}$ and $H = (h_{ij})_{n \times m}$ are bounded, that is, there exist some nonnegative constant $\tilde{w}_{ji}$, $\tilde{h}_{ij}$, such that $|w_{ji}| \leq \tilde{w}_{ji}$, $|h_{ij}| \leq \tilde{h}_{ij}$ for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$.

Remark 2.3. Currently, there exist some results for the synchronization of BAM neural networks when $c_i(x_i) = c_i x_i$, $d_j(y_j) = d_j y_j$ and matrices W^{T} and H are known in (2.1). For example, the asymptotical synchronization of BAM neural networks are discussed in [23,24]. Obviously, this property could not satisfy the situations when considering stochastic noise and unknown matrices W^{T} and H.

Lemma 2.4. *([25]). The following Jesen inequality*

$$\Big(\sum_{i=1}^{n} \xi_i^{\epsilon_2}\Big)^{1/\epsilon_2} \leq \Big(\sum_{i=1}^{n} \xi_i^{\epsilon_1}\Big)^{1/\epsilon_1}$$

holds for any positive constants $\xi_1, \xi_2, \cdots, \xi_n$ and $0 < \epsilon_1 < \epsilon_2$.

Lemma 2.5. *([26,27]). Let $x(t)$ be an n-dimensional Itô's process on $t \geq 0$ with the stochastic differential equation*

$$\mathrm{d}x(t) = f(t)\mathrm{d}t + g(t)\mathrm{d}\omega(t), \tag{2.4}$$

Let $V(x(t),t) \in C^{2,1}(\mathrm{R}^n \times \mathrm{R}^+; \mathrm{R}^+)$, then $V(x(t),t)$ is a real valued Itô's process with its stochastic differential given by

$$\mathrm{d}V(x(t),t) = \mathcal{L}V(x(t),t)\mathrm{d}t + V_x(x(t),t)g(t)\mathrm{d}\omega(t), \tag{2.5}$$

$$\mathcal{L}V(x(t),t) = V_t(x(t),t) + V_x(x(t),t)f(t) + \frac{1}{2}\mathrm{tr}\{g^{\mathrm{T}}(t)V_{xx}g(t)\}, \tag{2.6}$$

where

$$V_t(x(t),t) = \frac{\partial V(x(t),t)}{\partial t},$$

$$V_x(x(t),t) = \left(\frac{\partial V(x(t),t)}{\partial x_1}, \frac{\partial V(x(t),t)}{\partial x_2}, \cdots, \frac{\partial V(x(t),t)}{\partial x_n}\right)$$

and

$$V_{xx} = \left(\frac{\partial^2 V(x(t),t)}{\partial x_i \partial x_j}\right)_{n\times n}.$$

Moreover, if $f(0) = 0$, $g(0) = 0$, and there exist real numbers $\kappa > 0$ and $0 < \gamma < 1$, such that $V(x(t),t)$ satisfies

$$\mathcal{L}V(x(t),t) \le -\kappa V^{\gamma}(x(t),t), \tag{2.7}$$

then the origin of system (2.4) is stochastically stable in finite time interval $[t_0, T]$ for any $t_0 \ge 0$ and the initial condition $V(t_0) \ge 0$, moreover,

$$\mathbb{E}\{T\} = t_0 + \frac{V^{1-\gamma}(x(t_0),t_0)}{\kappa(1-\gamma)}.$$

3. MAIN RESULTS

In this section, we will discuss finite-time stochastic synchronization between systems (2.1) and (2.2) with the adaptive control method.

Firstly, we construct adaptive controller and updated laws as follows:

$$\begin{cases} p(t) = -\left(k_1 + \frac{1}{\|e_x^{\mathrm{T}}(t)\Lambda_1 e_x(t)\|_2} + \frac{k_2}{\|e_x^{\mathrm{T}}(t)\Lambda_1 e_x(t)\|_2^2}\right) e_x(t) - \hat{W}^{\mathrm{T}}(F(v(t)) \\ \qquad\quad -F(y(t))) - (k_3 - k_4(t))\,\mathrm{sign}(\Lambda_1 e_x(t)), \\ q(t) = -\left(k_1' + \frac{1}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2 e_y(t)\right\|_2} + \frac{k_2'}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2 e_y(t)\right\|_2^2}\right) e_y(t) - \hat{H}(G(u(t)) \\ \qquad\quad -G(x(t))) - (k_3' - k_5(t))\,\mathrm{sign}(\Lambda_2 e_y(t)), \\ \dot{\hat{W}}^{\mathrm{T}} = -2\beta_1\Lambda_1 e_x(t)F^{\mathrm{T}}(y(t)), \\ \dot{\hat{H}} = -2\beta_1\Lambda_2 e_y(t)G^{\mathrm{T}}(x(t)), \\ \dot{k}_4(t) = -\beta_2\left[\|\Lambda_1 e_x(t)\|_1 + \mathrm{sign}(k_4(t))\right], \\ \dot{k}_5(t) = -\beta_2\left[\|\Lambda_2 e_y(t)\|_1 + \mathrm{sign}(k_5(t))\right], \end{cases} \tag{3.1}$$

when $e_x(t) \neq 0$ or $e_y(t) \neq 0$, and $p(t) = q(t) = 0$ when $e_x(t) = e_y(t) = 0$, where $\Lambda_1 = \mathrm{diag}\{\lambda_1^1, \lambda_1^2 \cdots, \lambda_1^m\}, \Lambda_2 = \mathrm{diag}\{\lambda_2^1, \lambda_2^2 \cdots, \lambda_2^n\}, \lambda_1^i, \lambda_2^j, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n.$ are some positive constants to be determined.

Theorem 3.1. *Under the assumptions of* $(\mathrm{H}_1)-(\mathrm{H}_4)$, *if there exist positive definite diagonal matrices* $\Lambda_1=\mathrm{diag}(\lambda_1^1,\lambda_1^2,\cdots,\lambda_1^m)$, $\Lambda_2=\mathrm{diag}(\lambda_2^1,\lambda_2^2,\cdots,\lambda_2^n)$ *and scalars* k_i, k_i' $(i=1,2,3)$ *such that*

$$k_1>-\min_{1\le i\le m}\{c_i\},\quad k_1'>-\min_{1\le j\le n}\{d_j\},\tag{3.2}$$

$$k_3>a_1+a_2,\quad k_3'>b_1+b_2,\tag{3.3}$$

$$k_2+k_2'\ge\frac{1}{2}\delta_\theta+\sum_{i=1}^{n}\sum_{j=1}^{m}(|\hat{w}_{ji}|+\widetilde{w}_{ji})+\sum_{i=1}^{n}\sum_{j=1}^{m}(|\hat{h}_{ij}|+\widetilde{h}_{ij}),\tag{3.4}$$

where $\delta_\theta=(\delta_1+\delta_2)\theta_M$ *and* $\theta_M=\max\{\max\limits_{1\le i\le m}\lambda_1^i,\max\limits_{1\le j\le n}\lambda_2^j\}$, *then under the action of controller (3.1), the drive system (2.1) and the response system (2.2) can achieve finite-time stochastic synchronization and* $\mathbb{E}\{T\}=t_0+\beta^{-1}\sqrt{V(t_0)}$.

Proof. Consider the following Lyapunov function

$$\begin{aligned}V(t)=&\,e_x^{\mathrm{T}}(t)\Lambda_1e_x(t)+\frac{1}{2\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{w}_{ji}-w_{ji})^2+\frac{1}{\beta_2}k_4^2(t)\\&+e_y^{\mathrm{T}}(t)\Lambda_2e_y(t)+\frac{1}{2\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{h}_{ij}-h_{ij})^2+\frac{1}{\beta_2}k_5^2(t).\end{aligned}\tag{3.5}$$

By Itô differential formula, the differential of $V(t)$ along the error systems (2.3) gives

$$\begin{aligned}\mathrm{d}V(t)=&\,\mathcal{L}V(t)\mathrm{d}t+2e_x^{\mathrm{T}}(t)\Lambda_1(\varphi_2(t,u(t))-\varphi_1(t,x(t)))\mathrm{d}\omega\\&+2e_y^{\mathrm{T}}(t)\Lambda_2(\psi_2(v(t))-\psi_1(y(t)))\mathrm{d}\nu,\end{aligned}$$

where

$$\begin{aligned}\mathcal{L}V(t)=&V_t(x(t),t)+V_x(x(t),t)f(t)+\frac{1}{2}\mathrm{tr}\{g^{\mathrm{T}}(t)V_{xx}g(t)\}\\=&2e_x^{\mathrm{T}}\Lambda_1\dot{e}_x+\frac{1}{\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{w}_{ji}-w_{ji})\dot{\hat{w}}_{ji}+\frac{2}{\beta_2}k_4(t)\dot{k}_4(t)+\frac{2}{\beta_2}k_5(t)\dot{k}_5(t)+2e_y^{\mathrm{T}}\Lambda_2\dot{e}_y\\&+\frac{1}{\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{h}_{ij}-h_{ij})\dot{\hat{h}}_{ij}+M\\=&2e_x^{\mathrm{T}}\Lambda_1(-C(e_x(t))+\hat{W}^{\mathrm{T}}F(v(t))-W^{\mathrm{T}}F(y(t))+\Delta g_2(t,u(t))\\&-\Delta g_1(t,x(t))+p(t))\\&+2e_y^{\mathrm{T}}\Lambda_2(-D(e_y(t))+\hat{H}G(u(t))-HG(x(t))+\Delta h_2(t,v(t))\\&-\Delta h_1(t,y(t))+q(t))+M\\&+\frac{1}{\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{w}_{ji}-w_{ji})(-2\beta_1\lambda_1^ie_{x_i}(t)f_j(y_j(t)))\\&+\frac{2}{\beta_2}k_4(t)\left(-\beta_2\left[\|\Lambda_1e_x(t)\|_1+\mathrm{sign}(k_4(t))\right]\right)\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{h}_{ij}-h_{ij})(-2\beta_1\lambda_2^i e_{y_i}(t)g_j(x_j(t)))\\
&+\frac{2}{\beta_2}k_5(t)\left(-\beta_2\left[\|\Lambda_2 e_y(t)\|_1+\mathrm{sign}(k_5(t))\right]\right)\\
=&2e_x^{\mathrm{T}}\Lambda_1(-C(e_x(t))+\hat{W}^{\mathrm{T}}F(v(t))-W^{\mathrm{T}}F(y(t))+\Delta g_2(t,u(t))-\Delta g_1(t,x(t)))\\
&+2e_y^{\mathrm{T}}\Lambda_2(-D(e_y(t))+\hat{H}G(u(t))-HG(x(t))+\Delta h_2(t,v(t))-\Delta h_1(t,y(t)))\\
&+M\\
&-2\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{w}_{ji}-w_{ji})\lambda_1^i e_{x_i}(t)f_j(y_j(t))-2k_4(t)\left[\|\Lambda_1 e_x(t)\|_1+\mathrm{sign}(k_4(t))\right]\\
&-2\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{h}_{ij}-h_{ij})\lambda_2^i e_{y_i}(t)g_j(x_j(t))-2k_5(t)\left[\|\Lambda_2 e_y(t)\|_1+\mathrm{sign}(k_5(t))\right]\\
&-2e_x^{\mathrm{T}}\Lambda_1\left(k_1+\frac{1}{\|e_x^{\mathrm{T}}(t)\Lambda_1 e_x(t)\|_2}+\frac{k_2}{\|e_x^{\mathrm{T}}(t)\Lambda_1 e_x(t)\|_2^2}\right)e_x(t)\\
&-2e_x^{\mathrm{T}}\Lambda_1\hat{W}^{\mathrm{T}}(F(v(t))\\
&-F(y(t)))-2e_x^{\mathrm{T}}\Lambda_1(k_3-k_4(t))\mathrm{sign}(\Lambda_1 e_x(t))\\
&-2e_y^{\mathrm{T}}\Lambda_2\left(k_1'+\frac{1}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2 e_y(t)\right\|_2}+\frac{k_2'}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2 e_y(t)\right\|_2^2}\right)e_y(t)\\
&-2e_y^{\mathrm{T}}\Lambda_2\hat{H}(G(u(t))\\
&-G(x(t)))-2e_y^{\mathrm{T}}\Lambda_2\left(k_3'-k_5(t)\right)\mathrm{sign}(\Lambda_2 e_y(t)),
\end{aligned}
\tag{3.6}
$$

where $M=\mathrm{tr}\left((\varphi_2-\varphi_1)^{\mathrm{T}}\Lambda_1(\varphi_2-\varphi_1)\right)+\mathrm{tr}\left((\psi_2-\psi_1)^{\mathrm{T}}\Lambda_2(\psi_2-\psi_1)\right).$

It follows from assumptions (H_1) to (H_4), we have

$$-2e_x^{\mathrm{T}}\Lambda_1 C(e_x(t))\le -2\min_{1\le i\le m}(c_i)e_x^{\mathrm{T}}\Lambda_1 e_x(t), \tag{3.7}$$

$$-2e_y^{\mathrm{T}}\Lambda_2 D(e_y(t))\le -2\min_{1\le j\le n}(d_j)e_y^{\mathrm{T}}\Lambda_2 e_y(t), \tag{3.8}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{w}_{ji}-w_{ji})\lambda_1^i e_{x_i}(t)f_j(y_j(t))=e_x^{\mathrm{T}}\Lambda_1(\hat{W}^{\mathrm{T}}-W^{\mathrm{T}})F(y(t)), \tag{3.9}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{h}_{ij}-h_{ij})\lambda_2^i e_{y_i}(t)g_j(x_j(t))=e_y^{\mathrm{T}}\Lambda_2(\hat{H}-H)G(x(t)), \tag{3.10}$$

$$
\begin{aligned}
2e_x^{\mathrm{T}}\Lambda_1(\Delta g_2(t,u(t))-\Delta g_1(t,x(t)))&\le 2\left\|e_x^{\mathrm{T}}\Lambda_1\right\|_1\|\Delta g_2(t,u(t))-\Delta g_1(t,x(t))\|_\infty\\
&\le 2(a_1+a_2)\|\Lambda_1 e_x\|_1, \qquad (3.11)\\
2e_y^{\mathrm{T}}\Lambda_2(\Delta h_2(t,v(t))-\Delta h_1(t,y(t)))&\le 2\left\|e_y^{\mathrm{T}}\Lambda_2\right\|_1\|\Delta h_2(t,v(t))-\Delta h_1(t,y(t))\|_\infty\\
&\le 2(b_1+b_2)\|\Lambda_2 e_y\|_1, \qquad (3.12)
\end{aligned}
$$

$$M \leq (\delta_1 + \delta_2)\theta_M = \delta_\theta. \tag{3.13}$$

Noting that

$$\begin{aligned}2e_x^{\mathrm{T}}\Lambda_1(\hat{W}^{\mathrm{T}}F(v(t)) - W^{\mathrm{T}}F(y(t))) - 2e_x^{\mathrm{T}}\Lambda_1(\hat{W}^{\mathrm{T}} - W^{\mathrm{T}})F(y(t)))\\ = 2e_x^{\mathrm{T}}\Lambda_1\hat{W}^{\mathrm{T}}(F(v(t))-F(y(t))),\end{aligned} \tag{3.14}$$

and

$$\begin{aligned}2e_y^{\mathrm{T}}\Lambda_2(\hat{H}G(u(t))-HG(x(t)))-2e_y^{\mathrm{T}}\Lambda_2(\hat{H}-H)G(x(t)))\\ =2e_y^{\mathrm{T}}\Lambda_2\hat{H}(G(u(t))-G(x(t))).\end{aligned} \tag{3.15}$$

Substituting (3.7)-(3.15) into (3.6), we obtain

$$\begin{aligned}\mathcal{L}V(t) \leq& -2\min_{1\leq i\leq m}(c_i)e_x^{\mathrm{T}}\Lambda_1 e_x + 2(a_1+a_2)\left\|\Lambda_1 e_x\right\|_1 - 2k_1 e_x^{\mathrm{T}}\Lambda_1 e_x - 2\left\|e_x^{\mathrm{T}}\Lambda_1 e_x\right\|_2\\ &- 2\min_{1\leq j\leq n}(d_j)e_y^{\mathrm{T}}\Lambda_2 e_y + 2(b_1+b_2)\left\|\Lambda_2 e_y\right\|_1 - 2k_1' e_y^{\mathrm{T}}\Lambda_2 e_y - 2\left\|e_y^{\mathrm{T}}\Lambda_2 e_y\right\|_2\\ &- 2k_2 - 2k_2' - 2k_3\left\|\Lambda_1 e_x\right\|_1 - 2k_3'\left\|\Lambda_1 e_x\right\|_1 - 2|k_4(t)| - 2|k_5(t)| + \delta_\theta \qquad (3.16)\\ =&(-2\min_{1\leq i\leq m}(c_i) - 2k_1)e_x^{\mathrm{T}}\Lambda_1 e_x + 2(a_1+a_2-k_3)\left\|\Lambda_1 e_x\right\|_1 - 2\left\|e_x^{\mathrm{T}}\Lambda_1 e_x\right\|_2\\ &+ (-2\min_{1\leq j\leq n}(d_j) - 2k_1')e_y^{\mathrm{T}}\Lambda_2 e_y + 2(b_1+b_2-k_3')\left\|\Lambda_2 e_y\right\|_1 - 2\left\|e_y^{\mathrm{T}}\Lambda_2 e_y\right\|_2\\ &- 2k_2 - 2k_2' - 2|k_4(t)| - 2|k_5(t)| + \delta_\theta.\end{aligned}$$

Furthermore, by inequalities (3.2) and (3.3), one yields that

$$\mathcal{L}V \leq -2\left\|e_x^{\mathrm{T}}\Lambda_1 e_x\right\|_2 - 2\left\|e_y^{\mathrm{T}}\Lambda_2 e_y\right\|_2 - 2k_2 - 2k_2' - 2|k_4(t)| - 2|k_5(t)| + \delta_\theta, \tag{3.17}$$

Since

$$|\hat{w}_{ji}| + \tilde{w}_{ji} \geq |\hat{w}_{ji}| + |w_{ji}| \geq |\hat{w}_{ji} - w_{ji}|, |\hat{h}_{ij}| + \tilde{h}_{ij} \geq |\hat{h}_{ij}| + |h_{ij}| \geq |\hat{h}_{ij} - h_{ij}|.$$

by Lemma 1 and (3.4), we have

$$\begin{aligned}\mathcal{L}V \leq& -2\left\|e_x^{\mathrm{T}}\Lambda_1 e_x\right\|_2 - 2\left\|e_y^{\mathrm{T}}\Lambda_2 e_y\right\|_2 - 2k_2 - 2k_2' - 2|k_4(t)| - 2|k_5(t)| + \delta_\theta\\ &+ 2\sum_{i=1}^{n}\sum_{j=1}^{m}(|\hat{w}_{ji}| + \tilde{w}_{ji}) - 2\sum_{i=1}^{n}\sum_{j=1}^{m}|\hat{w}_{ji} - w_{ji}| + 2\sum_{i=1}^{n}\sum_{j=1}^{m}(|\hat{h}_{ij}| + \tilde{h}_{ij})\\ &- 2\sum_{i=1}^{n}\sum_{j=1}^{m}|\hat{h}_{ij} - h_{ij}|\\ \leq& -2\beta(\left\|e_x^{\mathrm{T}}\Lambda_1 e_x\right\|_2 + \frac{1}{\sqrt{2\beta_1}}\sum_{i=1}^{n}\sum_{j=1}^{m}|\hat{w}_{ji} - w_{ji}| + \frac{1}{\sqrt{\beta_2}}|k_4(t)|) \qquad (3.18)\\ &- 2\beta(\left\|e_y^{\mathrm{T}}\Lambda_2 e_y\right\|_2 + \frac{1}{\sqrt{2\beta_1}}\sum_{i=1}^{n}\sum_{j=1}^{m}|\hat{h}_{ij} - h_{ij}| + \frac{1}{\sqrt{\beta_2}}|k_5(t)|)\\ \leq& -2\beta\{e_x^{\mathrm{T}}\Lambda_1 e_x + \frac{1}{2\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{w}_{ji} - w_{ji})^2 + \frac{1}{\beta_2}k_4^2(t)\end{aligned}$$

$$
\begin{aligned}
&+e_y^{\mathrm{T}}\Lambda_2 e_y+\frac{1}{2\beta_1}\sum_{i=1}^{n}\sum_{j=1}^{m}(\hat{h}_{ij}-h_{ij})^2+\frac{1}{\beta_2}k_5^2(t)\}^{\frac{1}{2}}\\
=&-2\beta V^{\frac{1}{2}}(t).
\end{aligned}
$$

where $\beta=\min\{1,\sqrt{2\beta_1},\sqrt{\beta_2}\}$. Thus, it follows form Lemma 2, we have $\mathbb{E}\{\|e_x(t)\|_2\}=\mathbb{E}\{\|e_y(t)\|_2\}=0$ when $t\geq T$ and $\mathbb{E}\{T\}=t_0+\beta^{-1}\sqrt{V(t_0)}$. The proof is complete. $\square$

Remark 3.2. In fact, from the updated laws of (3.1), we can see that $\hat{W}^{\mathrm{T}}$ and $\hat{H}$ approach to some constants as $e_x(t)\to 0$ and $e_y(t)\to 0$. However, this does not mean that $\hat{W}^{\mathrm{T}}\to W^{\mathrm{T}}$ and $\hat{H}\to H$, as long as $\Delta g_2(t,u(t))\neq\Delta g_1(t,x(t))$ and $\Delta h_2(t,v(t))\neq\Delta h_1(t,y(t))$. This point is consistent with the results of [28].

Corollary 3.3. *Under the assumptions of* (H_1), (H_3) *and* (H_4), *further assume that there exist positive definite diagonal matrices* $\Lambda_1=\mathrm{diag}(\lambda_1^1,\lambda_1^2,\cdots,\lambda_1^m)$, $\Lambda_2=\mathrm{diag}(\lambda_2^1,\lambda_2^2,\cdots,\lambda_2^n)$ *and scalars* k_i, k_i' $(i=1,2)$ *such that*

$$
k_1>-\min_{1\leq i\leq m}\{c_i\},\qquad k_1'>-\min_{1\leq j\leq n}\{d_j\},\tag{3.19}
$$

$$
k_2+k_2'\geq\frac{1}{2}\delta_\theta+\sum_{i=1}^{n}\sum_{j=1}^{m}(|\hat{w}_{ji}|+\widetilde{w}_{ji})+\sum_{i=1}^{n}\sum_{j=1}^{m}(|\hat{h}_{ij}|+\widetilde{h}_{ij}),\tag{3.20}
$$

then under the action of adaptive controller and updated laws

$$
\begin{cases}
p(t)=-\left(k_1+\frac{1}{\|e_x^{\mathrm{T}}(t)\Lambda_1e_x(t)\|_2}+\frac{k_2}{\|e_x^{\mathrm{T}}(t)\Lambda_1e_x(t)\|_2^2}\right)e_x(t)-\hat{W}^{\mathrm{T}}(F(v(t))\\
\qquad -F(y(t)))+k_4(t)\mathrm{sign}(\Lambda_1e_x(t)),\\
q(t)=-\left(k_1'+\frac{1}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2e_y(t)\right\|_2}+\frac{k_2'}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2e_y(t)\right\|_2^2}\right)e_y(t)-\hat{H}(G(u(t))\\
\qquad -G(x(t)))+k_5(t)\mathrm{sign}(\Lambda_2e_y(t)),\\
\dot{\hat{W}}^{\mathrm{T}}=-2\beta_1\Lambda_1e_x(t)F^{\mathrm{T}}(y(t)),\\
\dot{\hat{H}}=-2\beta_1\Lambda_2e_y(t)G^{\mathrm{T}}(x(t)),\\
\dot{k}_4(t)=-\beta_2\left[\|\Lambda_1e_x(t)\|_1+\mathrm{sign}(k_4(t))\right],\\
\dot{k}_5(t)=-\beta_2\left[\|\Lambda_2e_y(t)\|_1+\mathrm{sign}(k_5(t))\right],
\end{cases}\tag{3.21}
$$

when $e_x(t)\neq 0$ *or* $e_y(t)\neq 0$, *and* $p(t)=q(t)=0$ *when* $e_x(t)=e_y(t)=0$, *where* $\delta_\theta=(\delta_1+\delta_2)\theta_M$ *and* $\theta_M=\max\{\max_{1\leq i\leq m}\lambda_1^i,\max_{1\leq j\leq n}\lambda_2^j\}$, *the drive systems (2.1) and the response systems (2.2) with* $\Delta g_1(t,x(t))=\Delta g_2(t,u(t))=\Delta h_1(t,y(t))=\Delta h_2(t,x(v))=0$ *achieve finite-time stochastic synchronization and* $\hat{W}^{\mathrm{T}}=W^{\mathrm{T}}$, $\hat{H}=H$ *when* $t\geq T$ *for any initial conditions. Moreover,* $\mathbb{E}\{T\}=t_0+\beta^{-1}\sqrt{V(t_0)}$.

Corollary 3.4. *Under the assumptions of* $(\mathrm{H}_1)-(\mathrm{H}_3)$, *further assume that there exist positive definite diagonal matrices* $\Lambda_1=\mathrm{diag}(\lambda_1^1,\lambda_1^2,\cdots,\lambda_1^m)$, $\Lambda_2=\mathrm{diag}(\lambda_2^1,\lambda_2^2,\cdots,\lambda_2^n)$ *and scalars* k_i, k_i' $(i=1,2,3)$ *such that*

$$
k_1>-\min_{1\leq i\leq m}\{c_i\},\quad k_1'>-\min_{1\leq j\leq n}\{d_j\},\tag{3.22}
$$

$$k_3 > a_1 + a_2, \quad k_3' > b_1 + b_2, \tag{3.23}$$

$$k_2 + k_2' \geq \frac{1}{2}\delta_\theta, \tag{3.24}$$

then under the action of adaptive controller and updated laws

$$\begin{cases} p(t) = -\left(k_1 + \frac{1}{\|e_x^{\mathrm{T}}(t)\Lambda_1 e_x(t)\|_2} + \frac{k_2}{\|e_x^{\mathrm{T}}(t)\Lambda_1 e_x(t)\|_2^2}\right) e_x(t) - W^{\mathrm{T}}(F(v(t)) \\ \qquad\quad -F(y(t))) - (k_3 - k_4(t))\,\mathrm{sign}(\Lambda_1 e_x(t)), \\ q(t) = -\left(k_1' + \frac{1}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2 e_y(t)\right\|_2} + \frac{k_2'}{\left\|e_y^{\mathrm{T}}(t)\Lambda_2 e_y(t)\right\|_2^2}\right) e_y(t) - H(G(u(t)) \\ \qquad\quad -G(x(t))) - (k_3' - k_5(t))\,\mathrm{sign}(\Lambda_2 e_y(t)), \\ \dot{k_4}(t) = -\beta_2\left[\|\Lambda_1 e_x(t)\|_1 + \mathrm{sign}(k_4(t))\right], \\ \dot{k_5}(t) = -\beta_2\left[\|\Lambda_2 e_y(t)\|_1 + \mathrm{sign}(k_5(t))\right], \end{cases} \tag{3.25}$$

when $e_x(t) \neq 0$ or $e_y(t) \neq 0$, and $p(t) = q(t) = 0$ when $e_x(t) = e_y(t) = 0$, where $\delta_\theta = (\delta_1 + \delta_2)\theta_M$ and $\theta_M = \max\{\max_{1\leq i\leq m}\{\lambda_1^i\}, \max_{1\leq j\leq n}\{\lambda_2^j\}\}, \tilde{\beta} = \min\{1, \sqrt{\beta_2}\}$, the drive systems (2.1) and the response systems (2.2) with the matrices $\hat{W}^{\mathrm{T}} = W$, $\hat{H} = H$ and the functions $\Delta g_1 = \Delta g_2$ realize finite time stochastic synchronization when $t \geq T$ for any initial conditions. Moreover, $\mathbb{E}\{T\} = t_0 + \tilde{\beta}^{-1}\sqrt{V(t_0)}$.

Remark 3.5. To the best of our knowledge, there is few results on finite-time stochastic synchronization for BAM neural networks with uncertain parameters. At the same time, the systems we investigated are very important BAM neural networks which have good applications perspective. Further, the proposed controller and updated laws in this paper are simpler one.

4. SIMULATION RESULTS

In this section, a numerical simulation is performed to verify the feasibility and effectiveness of the above synchronization criteria. In the numerical simulation, we use the Euler-Maruyama numerical scheme for stochastic differential equations.

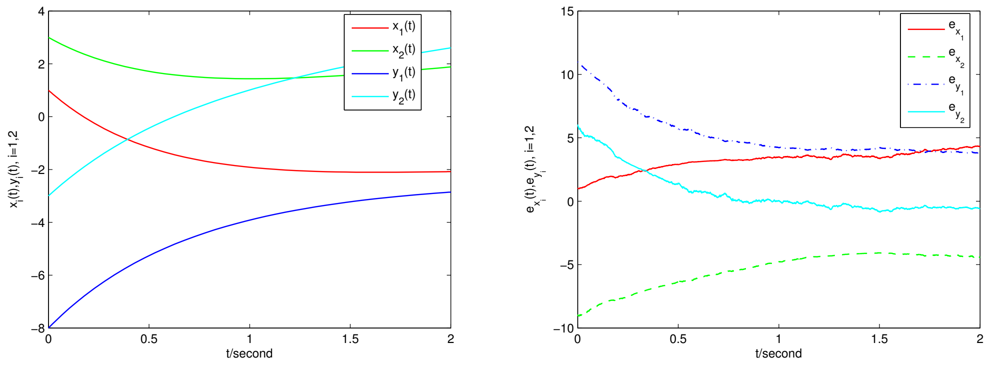
Example 4.1. Consider the following BAM neural network model as

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1.2 & -0.2 \\ -0.2 & 1.1 \end{bmatrix}\begin{bmatrix} f_1(y_1(t) \\ f_2(y_2(t) \end{bmatrix} + \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \\ \begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0.8 & -0.35 \\ -0.35 & 1.1 \end{bmatrix}\begin{bmatrix} g_1(x_1(t) \\ g_2(x_2(t) \end{bmatrix} + \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}, \end{cases} \tag{4.1}$$

where $c_i(x_i) = x_i$, $d_j(y_j) = y_j$, $f_j(y_j) = \frac{3}{4}y_j$, $g_i(x_i) = x_i$, $i, j = 1, 2$. Let $I_1 = 1$, $I_2 = 0$, $J_1 = 0$, $J_2 = 1$, then state trajectories of (4.1) are shown in Fig.1(a). The other parameters in the drive systems (2.1) are chosen as follows:

$$\Delta g_1(t, x(t)) = (\tanh(0.4x_1(t)), \tanh(0.7x_2(t)))^{\mathrm{T}},$$

$$\varphi_1(t, x(t)) = \mathrm{diag}\{0.2\sin(x_1(t)), \cos(x_2(t))\},$$



(a) (b)

Fig. 1: (a)The state trajectories of (4.1).(b)The state trajectories of error system (2.3) without controller (3.1).

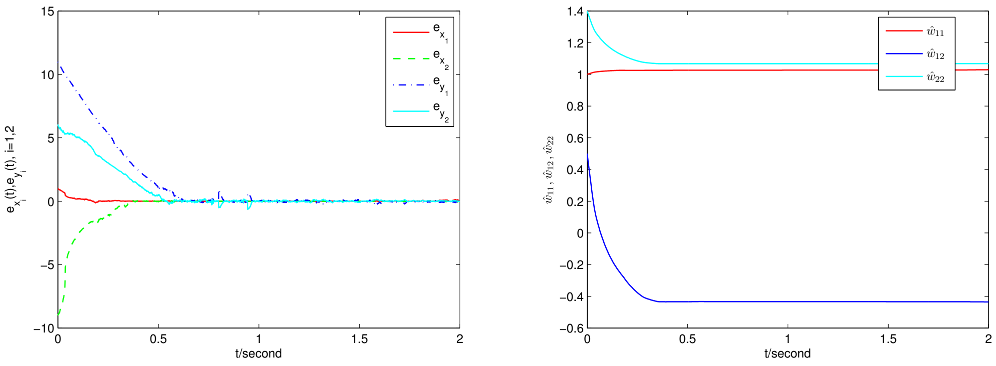
$$\Delta h_1(t, y(t)) = (-\tanh(0.2y_2(t)), \tanh(0.5y_1(t)))^{\mathrm{T}},$$
$$\psi_1(t, y(t)) = \mathrm{diag}\{-0.2\cos(y_2(t)), \sin(y_1(t))\}.$$

In the response system (2.2),we take

$$\Delta g_2(t, u(t)) = (\tanh(0.6u_2(t)), \tanh(0.3u_1(t)))^{\mathrm{T}},$$
$$\varphi_2(t, u(t)) = \mathrm{diag}\{0.5\cos(u_2(t)), -0.4\sin(u_1(t))\},$$
$$\Delta h_2(t, v(t)) = (-\tanh(0.8v_1(t)), \tanh(0.2v_2(t)))^{\mathrm{T}},$$
$$\psi_2(t, v(t)) = \mathrm{diag}\{0.8\sin(v_1(t)), 0.2\sin(v_2(t))\}.$$

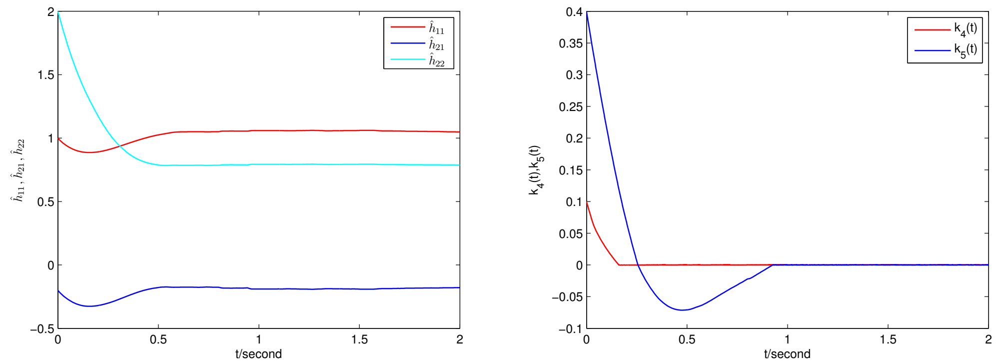
It is easy to validate that the assumptions $(\mathrm{H}_1) - (\mathrm{H}_3)$ hold if taking $a_i = b_i = 1$ and $\delta_i = 2.5, i = 1, 2$. Then the state trajectories of error systems (2.3) without controller (3.1) are shown in Fig.1(b) which does not exhibit synchronization. After computing, we obtained the feasible solutions of $(3.2) - (3.4)$ as $\Lambda_1 = \mathrm{diag}(0.2, 0.5)$, $\Lambda_2 = \mathrm{diag}(0.4, 0.8)$, $k_1 = k_1' = 1$, $k_2 = k_2' = 10.5$ and $k_3 = k_3' = 3$.

Therefore, it follows from Theorem 1 that the drive systems (2.1) and the response systems (2.2) with the given parameters can realize finite time stochastic synchronization. Letting $\beta_1 = \beta_2 = 0.2$ and the initial conditions $x(0) = (1, 3)^{\mathrm{T}}, y(0) = (-8, -3)^{\mathrm{T}}$, $u(0) = (2, -6)^{\mathrm{T}}, v(0) = (3, 3)^{\mathrm{T}}$, then the state trajectories of error system (2.3) and matrices $\hat{W}^{\mathrm{T}}$, $\hat{H}$ are shown in Fig.2-3. In particular, Fig.2(a) shows that the drive system (2.1) synchronizes with the response system (2.2) in finite time $T = 1$. That is, Fig.2(a) strongly supports the conclusion. Fig.2(b) and Fig.3(a) show that unknown parameters trajectories of $\hat{W}^{\mathrm{T}}$ and $\hat{H}$, where we assume $\hat{w}_{21} = -0.2$ and $\hat{h}_{12} = -0.35$ are known in prior. Fig.3(b) is the evolution of control gains k_4 and k_5 in (3.1).



(a) (b)

Fig. 2: (a)The state trajectories of error system (2.3) with controller (3.1).(b)The trajectories of the identified element of matrix $\hat{W}^{\mathrm{T}}$ in (3.1).



(a) (b)

Fig. 3: (a)The trajectories of the identified element of matrix $\hat{H}$ in (3.1).(b)Evolutions of control gains k_4 and k_5 in (3.1).

5. CONCLUSION

This paper is concerned with the finite-time stochastic synchronization for a class of BAM neural networks with uncertain parameters. Based on stochastic differential equations stability theory, by designing adaptive controller and updated laws, sufficient conditions are obtained to ensure finite-time stochastic synchronization and the unknown parameters be identified in finite-time for the considered systems. Moreover, we also provide numerical simulations to support the theoretical result. Our results in this paper complement and extend some existing results. In real world, for large scale BAM neural networks, time delays is unavoidable and should be taken into account due to the finite information transmission and processing speed among the network nodes. Therefore, finite-time stochastic synchronization of BAM neural networks with uncertain parameters and delays is our next research topic.

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