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OPTIMAL INVENTORY MODEL FOR DETERIORATING PRODUCTS WITH WEIBULL DISTRIBUTION DETERIORATION, STOCK AND TIME-DEPENDENT DEMAND, TIME-VARYING HOLDING COST UNDER FULLY BACKLOGGING

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ABSTRACT. Effect of deterioration plays a vital role in present environment of market. In this paper, an optimal inventory model for deteriorating items having stock and time dependent demand under the effects of deterioration is studied. A two parameter Weibull distribution is used to represent the distribution of the time to deterioration. In which the shortages are allowed and fully backlogged. The model is solved analytically by minimizing the total inventory cost. Some numerical examples have been carried out to illustrate the developed model.

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1. INTRODUCTION

Many mathematical models have been developed to control the inventory for smooth and efficient running business. Most of the earlier inventory models consider that the demand rate as well as deterioration rate is constant but this is the feature of static environment. While now a days market depends on various factors which effect the economy of the business. Thus the market is fluctuating day by day and in such a dynamic environment nothing is fixed or constant.

Deterioration is defined a decay, damage, spoilage evaporation and loss of unity of the product. Deterioration in inventory is a realistic feature and need to consideration it. Often we encounter products such as fruits, milk, drug, vegetables and photographic films etc that have a defined period of life time. Such items are referred as deterioration items. The loss due to deterioration cannot be avoided. Due to deterioration, inventory system faces the problem of shortages and loss of good will or loss of profit.

Dependent demand order quantities are computed using a system called material requirements planning (MRP), which considers not only the quantities of each of the component parts needed, but also the lead times needed to produce and receive the

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items. In the classical inventory models, the demand rate is regularly assumed to be either constant or time-dependent but independent of the stock levels. However, practically an increase in shelf space for an item induces more consumers to buy it. This occurs owing to its visibility, popularity or variety. Conversely, low stocks of certain goods might raise the perception that they are not fresh. Therefore, it is observed that the demand rate may be influenced by the stock levels for some certain types of inventory. In years, marketing researchers and practitioners have recognized the phenomenon that the demand for some items could be based on the inventory level on display.

The most obvious holding costs include rent for the required space; equipment, materials, and labor to operate the space; insurance; security; interest on money invested in the inventory and space, and other direct expenses. Some stored goods become obsolete before they are sold, reducing their contribution to revenue while having no effect on their holding cost. Some goods are damaged by handling, weather, or other mechanisms. Some goods are lost through mishandling, poor record keeping, or theft, a category euphemistically called shrinkage.Holding cost also includes the opportunity cost of reduced responsiveness to customers' changing requirements, slowed introduction of improved items, and the inventory's value and direct expenses, since that money could be used for other purposes.

A shortage is a situation in which demand for a good or service exceeds the available supply. Possible causes of a shortage include miscalculation of demand by a company producing a good or service, resulting in the inability to keep up with demand, or government policies such as price fixing or rationing. Natural disasters that devastate the physical landscape of a region can also cause shortages of such essential products as food and housing, also leading to higher prices of those goods.

A flexible measurement that details the probable distribution associated with the lifetime characteristics of a particular part or service component. Particularly focused on failure-rate, Weibull distribution is used throughout reliability engineering to anticipate and account for issues of wear-out during development. Here we assumed an optimal inventory model for deteriorating products with Weibull distribution deterioration.

2. LITERATURE REVIEW

In [1] Amutha and Chandrasekaran presented a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model that minimize the total inventory cost. In [8] Krishna and Bani considered a mathematical model of an inventory system in which demand depending upon stock level and time with various degree. and developed that failure rate and life expectancy of many item can be expressed in terms of Weibull distribution. In [16] Singh and Malik proposed a deterministic two warehouses inventory model for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. [7] Karuppasamy and Uthayakumar assumed a model for defective item with time dependent demand, holding cost and give analytical solution of the model that minimize the total inventory cost. [9] Kiransinh and Pravin presented of an inventory system with stock dependent demand, in which the holding cost is a step function of storage time and variable ordering cost. [3] Chun discussed why it is appropriate maximize the profits, instead of minimizing the costs, in an inventory system with an inventory level dependent demand rate. [4] Diwakar et al. presented an inventory model considering the demand as a parametric dependent linear function of time and price both. The coefficient of time parameter and coefficient of price parameter are examined simultaneously and proved that time is dominating variable over price in terms of earning more profit. [5] Hesham and Ahmed presented an inventory model with a variable demand rate, a variable holding cost, and a variable purchase cost.

In [20] Vinod and Lal developed the deterioration factor taken into either direct spoilage or physical decay in the course time, deterioration is natural feature in the inventory system. In [21] Vinod and Lal described a model for deteriorating item with time dependent demand and partial backlogging and give analytical solution of the model. In [22] Zeinab et al. developed an inventory model for a main class of deteriorating items, namely perishable products, under stochastic lead time assumption. And proposed model for a uniform distribution function that could be tractable to solve optimally by means of an exact approach. [10] Mishra et al. derived for perishable items with time dependent demand pattern. And time proportional deterioration rate is used. In this model,shortages are allowed. [11] Mohan and Venkateswarlu developed inventory management models for deteriorating items when the demand rate is assumed to be linear function of time. And assumed that the deterioration rate is proportional to time. [6] Jie et al. developed a lotsizing model for deteriorating items with a current stock dependent demand and delay in payments.And provided the necessary and sufficient conditions of the existence and uniqueness of the optimal solutions that could maximize the retailers average profit per unit time.

In [17] Sunil and Pravin discussed Weibull distribution is considered for deterioration with time varying holding cost, shortage are permitted and kept backlogged. [18] Trailokyanath and Hadibandhu focused on determining the EOQ (Economic Order Quantity) model of an inventory system for a deteriorating item. Deterioration considered Weibull distribution deterioration rate when the demand is taken as the trapezoidal function of time. [12] Pareek and Sharma developed for Weibull Distribution deteriorating items with exponential declining demand and the shortages are partially backlogged. Also showed that show that the minimized objective cost function is jointly convex and derive the optimal solution. [14] Sharma et al. developed for deteriorating items that deteriorates at a Weibull distributed rate, assuming the demand rate as a ramp type function of time. [19] Vikas and Rekha developed an inventory model for the rate of deterioration follows the Weibull distribution with two parameters. [13] Sarkar and Chakrabarti present paper an EPQ model of deteriorating items having weibull distribution deterioration has been studied with permissible delay in payments. [2] Babu and Nagarani studied an inventory model for two parameter Weibull distribution deterioration with selling price demand rate, in which shortages are allowed and are partially backlogged.

3. NOTATIONS AND ASSUMPTIONS

3.1. NOTATIONS.

A	the ordering cost per order
C	the purchasing cost per order
a_1	the deterioration rate.
$h(t)$	the inventory carrying cost per item
π_b	the back ordered cost per unit short per time unit
t_1	the time at which the inventory level reaches zero; $t_1 \geq 0$
t_2	the length of period during which shortages are allowed; $t_2 \geq 0$
T	$(= t_1 + t_2)$ the length of the cycle time
IM	the maximum inventory level during $[0, T]$
IB	the maximum inventory level during shortage period
Q	$(= IM + IB)$ the order quantity during a cycle of length T
$q_1(t)$	the level of positive inventory at time t
$q_2(t)$	the level of negative inventory at time t
$TC(t_1, t_2)$	the total cost per time unit

3.2. ASSUMPTION.

1. Demand rate is defined as the function of stock and time as
$$D(q, t) = a + bt^{\beta-1}q(t).$$
2. The items considered in this model are deteriorating with time.
3. The deterioration rate is defined as two parameter Weibull distribution $a_1(t) = \alpha\beta t^{\beta-1}$, where $0 < \alpha < 1$.
4. Shortages are allowed and completely backlogged.
5. T is the duration of a cycle.
6. Lead time is zero and replenishment is instantaneous.

7. The planning horizon is finite.

4. **PROPOSED MODEL**

The rate of change of inventory during positive stock period $[0, t_1]$ and shortage period $[t_1, T]$ is governed.

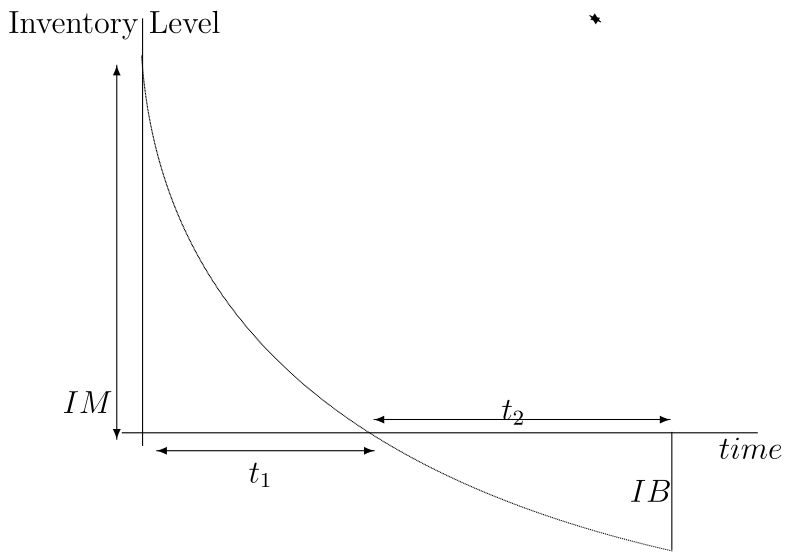

FIGURE 1. Graphical representation of inventory system

5. **MATHEMATICAL FORMULATION**

The inventory level q(t) changes with respect to time t due to the effects of demand as well as deterioration hence the variation of inventory level q(t) with time t is as follows:

$$\frac{dq_1}{dt} = -(\alpha\beta)t^{\beta-1}q_1(t) - (a + bt^{\beta-1})q_1(t), \quad 0 \leq t \leq t_1, \tag{5.1}$$

$$\frac{dq_2(t)}{dt} = -(a + bt^{\beta-1}q_2(t)), \quad t_1 \leq t \leq T \tag{5.2}$$

with boundary conditions

$$q(0) = IM \text{ at } t = 0, q(t_1) = 0 \text{ at } t = t_1 \ \ and \ q(T) = IB \text{ at } t = T. \tag{5.3}$$

Using the assumption that $D(q, t)$, the demand function and $a_1(t)$, the deterioration rate as a non linear function

$$D(q, t) = a + bt^{\beta-1}q(t) \ \ and \ a_1(t) = \alpha\beta t^{\beta-1} \ then(5.1), (5.2)reduce \ to$$

$$\frac{dq_1}{dt} + (\alpha\beta + b)t^{\beta-1}q_1(t) = -a, 0 \leq t \leq t_1 \tag{5.4}$$

$$\frac{dq_2(t)}{dt} + bt^{\beta-1}q_2(t) = -a, t_1 \leq t \leq T \tag{5.5}$$

6. SOLUTION PROCEDURE

6.1. CASE (a): $\beta > 1$ — WHEN THE DETERIORATION RATE IS DIRECTLY PROPORTIONAL TO TIME.

Solution of the above differential equation (5.4)and (5.5) with the boundary condition

$$q_1(t) = ae^{-kt^\beta}((t_1 - t) + \frac{k}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}),\ 0 \le t \le t_1 \text{ where } k = \frac{\alpha\beta + b}{\beta} \quad (6.1)$$

$$q_2(t) = ae^{-\frac{bt^\beta}{\beta}}\left[(t_1 - t) + \frac{b}{\beta(\beta+1)}(t_1^{\beta+1} - t^{\beta+1})\right], \quad t_1 \le t \le T. \quad (6.2)$$

Using the boundary conditions(5.3), Equations(6.1) and (6.2) gives the value of maximum inventory level IM and IB as below

$$IM = a\left(t_1 + \frac{kt_1^{\beta+1}}{\beta+1}\right), \quad (6.3)$$

$$IB = ae^{-\frac{bt^\beta}{\beta}}\left[(t_1 - T) + \frac{b}{\beta(\beta+1)}(t_1^{\beta+1} - T^{\beta+1})\right], \quad (6.4)$$

From(6.3) and (6.4) ordered quantity for the next replenishment can be expressed as

$$Q = IM + IB,$$

$$Q = a\left(t_1 + \frac{kt_1^{\beta+1}}{\beta+1}\right) + ae^{-\frac{bt^\beta}{\beta}}\left[(t_1 - T) + \frac{b}{\beta(\beta+1)}(t_1^{\beta+1} - T^{\beta+1})\right]. \quad (6.5)$$

Therefore the total cost per replenishment cycle consists of the following components:

1. Inventory Holding Cost Per Cycle

$$IHC = \int_0^{t_1} h(t)q_1(t)dt, \quad (6.6)$$

$$IHC = a\left[x\left(\frac{t_1^2}{2} + \frac{kt_1^{\beta+2}}{\beta+2} - \frac{kt_1^{\beta+2}}{\beta+1} - \frac{k^2t_1^{2(\beta+1)}}{(\beta+1)^2}\right) + y\left(\frac{t_1^3}{6} - \frac{kt_1^{\beta+3}}{\beta+2} + \frac{kt_1^{\beta+3}}{(\beta+1)^2} - \frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}\right)\right]. \quad (6.7)$$

2. Back-Ordered Cost Per Cycle

$$BC = \pi_b \int_{t_1}^{T} -q_2(t)dt$$

$$\begin{aligned} BC &= a\pi_b\Bigg[t_1T-\frac{T^2}{2}+\frac{b}{\beta(\beta+1)}t_1^{\beta+1}T-\frac{bT^{\beta+2}}{\beta(\beta+1)(\beta+2)}-\frac{bT^{\beta+1}t_1}{\beta(\beta+1)} \\ &\quad+\frac{bT^{\beta+2}}{\beta(\beta+2)}+\frac{b^2}{\beta(\beta+1)}\left(\frac{T^{\beta+1}t_1^{\beta+1}}{\beta+1}\right)\frac{b^2T^{\beta+3}}{\beta(\beta+1)(\beta+3)}-\frac{t_1^2}{2}+\frac{bt_1^{\beta+2}}{\beta(\beta+1)(\beta+2)} \\ &\quad-\frac{bt_1^{\beta+2}}{\beta(\beta+2)}-\frac{b^2t_1^{2(\beta+1)}}{\beta(\beta+1)^2}+\frac{b^2t_1^{\beta+3}}{\beta(\beta+1)(\beta+3)}\Bigg]. \end{aligned} \tag{6.8}$$

3. Purchase Cost Per Cycle

Purchase cost per cycle

$$= \Big(\text{Purchase cost per unit}\Big) * \Big(\text{Order quantity in one cycle}\Big),$$

$$PC = C * Q,$$

$$PC = Ca\left(t_1+\frac{kt_1^{\beta+1}}{\beta+1}\right)+ae^{-\frac{bt^{\beta}}{\beta}}\left[(t_1-T)+\frac{b}{\beta(\beta+1)}(t_1^{\beta+1}-T^{\beta+1})\right]. \tag{6.9}$$

4. Ordering Cost

$$OC = A$$

6.2. **CASE (b):** $\beta=\frac{1}{n}$ **— WHEN THE DETERIORATION RATE IS INVERSELY PROPORTIONAL TO TIME OR CONSTANT.** Solution of the above differential equation (5.4) and (5.5) with boundary condition (5.3) are obtained as bellow:

$$q_1(t)=\frac{a}{k\beta}\left(-\frac{1}{k}\right)^{\frac{1}{\beta}-1}e^{-kt^{\beta}}\left[I_{\frac{1}{\beta}}-(I_{\frac{1}{\beta}})_{t=t_1}\right]. \tag{6.10}$$

Here

$$I_{\frac{1}{\beta}}=-\Big(-kt^{\beta}\Big)^{\frac{1}{\beta}-1}e^{kt^{\beta}}-\left(\frac{1}{\beta}-1\right)\Big(-kt^{\beta}\Big)^{\frac{1}{\beta}-2}e^{kt^{\beta}},\qquad k=\frac{\alpha\beta+b}{\beta},$$

$$q_2(t)=\frac{a}{k_1\beta}\left(-\frac{1}{k_1}\right)^{\frac{1}{\beta}-1}e^{-kt^{\beta}}\left[I'_{\frac{1}{\beta}}-(I'_{\frac{1}{\beta}})_{t=t_1}\right], \tag{6.11}$$

and

$$I'_{\frac{1}{\beta}}=-\Big(-k_1t^{\beta}\Big)^{\frac{1}{\beta}-1}e^{k_1t^{\beta}}-\left(\frac{1}{\beta}-1\right)\Big(-k_1t^{\beta}\Big)^{\frac{1}{\beta}-2}e^{k_1t^{\beta}},\quad k_1=\frac{b}{\beta}.$$

6.3. **SPECIAL CASE 1:** β **=1.**

$$q_1(t) = ae^{kt}\left((t_1 - t) + \frac{k}{2}(t_1^2 - t^2)\right), \quad 0 \le t \le t_1,$$

$$q_2(t) = ae^{-bt}\left((t_1 - t) + \frac{b}{2}(t_1^2 = t^2)\right), \quad t_1 \le t \le T,$$

$$IHC = a\left(x\left(\frac{t_1^2}{2} - \frac{kt_1^3}{6} - \frac{kt_1^4}{4}\right) + y\left(\frac{t_1^3}{6} - \frac{kt_1^4}{3}\right)\right),$$

$$BC = a\pi_b\left[t_1 T - \frac{T^2}{2} + \frac{bt^2 T}{2} + \left(\frac{bT62}{6}\right) - \frac{bT^2 t_1}{2} + \frac{b^2 T^2 t_1^2}{4} - \frac{b^{2T^4}}{8} - \frac{t_1^2}{2} + \frac{bt_1^2}{6} - \frac{bt_1^3}{3} - \left(\frac{b^2 t_1^4}{8}\right)\right],$$

$$PC = Ca\left(t_1 + \frac{kt_1^2}{2} + e^{-bT}\left((t_1 - t) + \frac{b}{2}\left(t_1^2 - T^2\right)\right)\right).$$

6.4. **SPECIAL CASE 2:** $\beta = \frac{1}{2}$**.** Solutions (6.10) and (6.11) reduces to

$$q_1(t) = \frac{2a}{k^2}e^{-k\sqrt{t}}\left[\left(k\sqrt{t_1} - 1\right)e^{k\sqrt{t_1}} - \left(k\sqrt{t} - 1\right)e^{k\sqrt{t}}\right], \quad 0 \le t \le t_1, \tag{6.12}$$

$$q_2(t) = \frac{ae^{-2b\sqrt{t}}}{2b^2}\left[\left(2b\sqrt{t_1} - 1\right)e^{2b\sqrt{t_1}} - \left(2b\sqrt{t} - 1\right)e^{2b\sqrt{t}}\right]. \tag{6.13}$$

7. OPTIMALITY OF THE COST FUNCTION

The total cost per time unit is given by

$$TC = \frac{1}{T}\left[OC + IHC + BC + PC\right],$$

$$\begin{aligned} TC =& \frac{1}{T}\Bigg[A + a\Bigg[x\left(\frac{t_1^2}{2} + \frac{kt_1^{\beta+2}}{\beta+2} - \frac{kt_1^{\beta+2}}{\beta+1} - \frac{k^2 t_1^{2(\beta+1)}}{(\beta+1)^2}\right) + \\ & y\left(\frac{t_1^3}{6} - \frac{kt_1^{\beta+3}}{\beta+2} + \frac{kt_1^{\beta+3}}{(\beta+1)^2} - \frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}\right)\Bigg] + a\pi_b\Bigg[t_1 T - \frac{T^2}{2} + \frac{b}{\beta(\beta+1)}t_1^{\beta+1}T \\ & - \frac{bT^{\beta+2}}{\beta(\beta+1)(\beta+2)} - \frac{bT^{\beta+1}t_1}{\beta(\beta+1)} + \frac{bT^{\beta+2}}{\beta(\beta+2)} + \frac{b^2}{\beta(\beta+1)} \\ & \left(\frac{T^{\beta+1}t_1^{\beta+1}}{\beta+1}\right) - \frac{b^2 T^{\beta+3}}{\beta(\beta+1)(\beta+3)} - \frac{t_1^2}{2} + \frac{bt_1^{\beta+2}}{\beta(\beta+1)(\beta+2)} \\ & - \frac{bt_1^{\beta+2}}{\beta(\beta+2)} - \frac{b^2 t_1^{2(\beta+1)}}{\beta(\beta+1)^2} + \frac{b^2 t_1^{\beta+3}}{\beta(\beta+1)(\beta+3)}\Bigg] \\ & + Ca\left[\left(t_1 + \frac{kt_1^{\beta+1}}{\beta+1}\right) + ae^{-\frac{bt^\beta}{\beta}}\left[(t_1 - T) + \frac{b}{\beta(\beta+1)}(t_1^{\beta+1} - T^{\beta+1})\right]\right]. \end{aligned} \tag{7.1}$$

Differentiating the cost function with respect to t_1 and T

$$\frac{\partial TC}{\partial t_1} = \frac{1}{T}\Bigg[a\Bigg[x\Bigg(t_1 + \frac{(\beta+2)kt_1^{\beta+1}}{\beta+2} - \frac{k(\beta+2)t_1^{\beta+1}}{\beta+1} - \frac{k^2(2\beta+2)t_1^{2\beta+1}}{(\beta+1)^2}\Bigg)$$
$$+y\Bigg(\frac{t_1^2}{3} - \frac{(\beta+3)kt_1^{\beta+2}}{\beta+2} + \frac{(\beta+3)kt_1^{\beta+2}}{(\beta+1)^2} - \frac{kt_1^{\beta+2}}{(\beta+1)}\Bigg)\Bigg] + a\pi_b\Bigg[\frac{bt_1^{\beta}T}{\beta(\beta+1)}$$
$$-\frac{bT^{\beta+1}}{\beta(\beta+1)} + b^2T^{\beta+1}t_1^{\beta} - t_1^2 + \frac{bt_1^{\beta+1}}{\beta(\beta+1)} - \frac{bt_1^{\beta+1}}{\beta}\frac{b^2(2(\beta+1)t_1^{2\beta+1}}{\beta(\beta+1)^2} + \frac{b^2t_1^{\beta+2}}{\beta(\beta+1)}\Bigg]\Bigg],$$

$$\frac{\partial TC}{\partial T} = \frac{1}{T}\Bigg[a\pi_b\Bigg[t_1 - T + \frac{bt_1^{\beta+1}}{\beta(\beta+1)} - \frac{bT^{\beta}t_1}{\beta(\beta+1)} + \frac{bT^{\beta+1}}{\beta} + b^2T^{beta}t_1\beta+1 - \frac{b^2T^{\beta+2}}{\beta(\beta+1)}\Bigg]\Bigg]$$
$$+Ca\Bigg[ae^{\frac{-bt_1^{\beta}}{\beta}}\Bigg(-1 - \frac{bt_1^{\beta}}{\beta}\Bigg)\Bigg] + \frac{1}{T^2}\Bigg[A + a\Bigg[x\Bigg(\frac{t_1^2}{2} + \frac{kt_1^{\beta+2}}{\beta+2} - \frac{kt_1^{\beta+2}}{\beta+1} - \frac{k^2t_1^{2(\beta+1)}}{(\beta+1)^2}\Bigg) +$$
$$y\Bigg(\frac{t_1^3}{6} - \frac{kt_1^{\beta+3}}{\beta+2} + \frac{kt_1^{\beta+3}}{(\beta+1)^2} - \frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}\Bigg)\Bigg]$$
$$+a\pi_b\Bigg[t_1T - \frac{T^2}{2} + \frac{b}{\beta(\beta+1)}t_1^{\beta+1}T$$
$$-\frac{bT^{\beta+2}}{\beta(\beta+1)(\beta+2)} - \frac{bT^{\beta+1}t_1}{\beta(\beta+1)} + \frac{bT^{\beta+2}}{\beta(\beta+2)} + \frac{b^2}{\beta(\beta+1)}$$
$$\Bigg(\frac{T^{\beta+1}t_1^{\beta+1}}{\beta+1}\Bigg) - \frac{b^2T^{\beta+3}}{\beta(\beta+1)(\beta+3)} - \frac{t_1^2}{2} + \frac{bt_1^{\beta+2}}{\beta(\beta+1)(\beta+2)}$$
$$-\frac{bt_1^{\beta+2}}{\beta(\beta+2)} - \frac{b^2t_1^{2(\beta+1)}}{\beta(\beta+1)^2} + \frac{b^2t_1^{\beta+3}}{\beta(\beta+1)(\beta+3)}\Bigg]$$
$$+Ca\Bigg[\Bigg(t_1 + \frac{kt_1^{\beta+1}}{\beta+1}\Bigg) + ae^{-\frac{bt^{\beta}}{\beta}}\Bigg[(t_1 - T) + \frac{b}{\beta(\beta+1)}(t_1^{\beta+1} - T^{\beta+1})\Bigg]\Bigg].$$

The optimal values of t_1 and T can be obtained by satisfying the necessary condition for minimization of the cost function

$$\frac{\partial TC}{\partial t_1} = 0, \frac{\partial TC}{\partial T} = 0 \tag{7.2}$$

provided the following sufficient conditions are satisfied

$$\frac{\partial^2 TC}{\partial t_1^2} > 0, \frac{\partial^2 TC}{\partial T^2} > 0 \tag{7.3}$$

$$\Bigg(\frac{\partial^2 TC}{\partial t_1^2}\Bigg)\Bigg(\frac{\partial^2 TC}{\partial T^2}\Bigg) - \Bigg(\frac{\partial^2 TC}{\partial t_1 \partial T}\Bigg)^2 > 0 \tag{7.4}$$

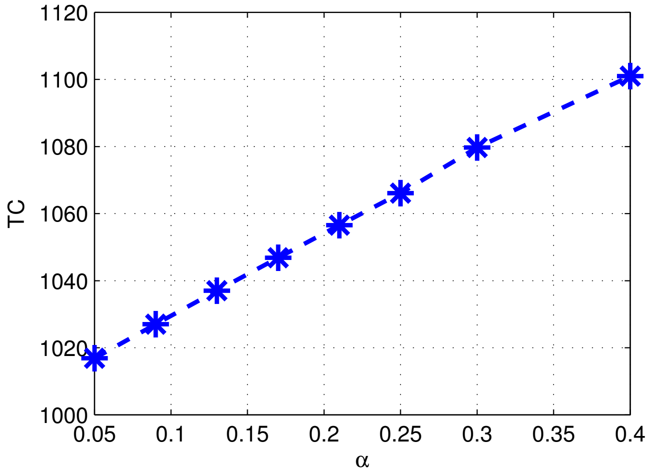

FIGURE 2. Variation of optimal cost TC with respect to deterioration parameter α

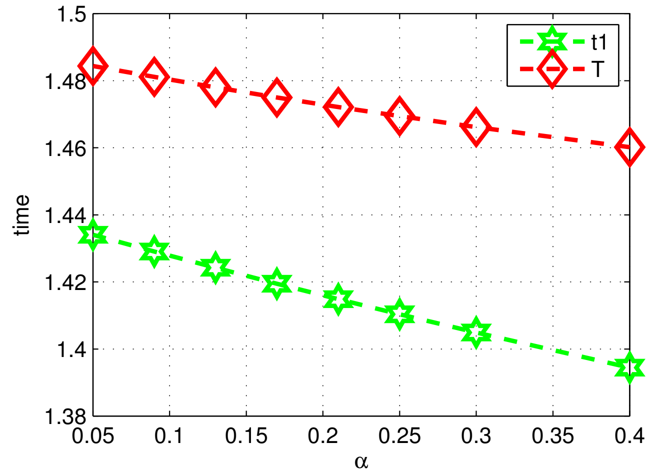

FIGURE 3. Variation of optimal time t_1 and T with respect to deterioration parameter α

8. NUMERICAL RESULTS AND OBSERVATION

We incorporated model verification as a special case 6.3. For this special case demand depends on current stock level only and item deteriorates with constant rate of deterioration. Solution(6.1) and (6.2) for the special case 6.4 are in good agreement with the solution obtained by [15] Singh et al. which gives the strong validation and more generalization of our proposed model. However we also incorporated the more practical numerical example for the support of our model verification.

Numerical values of t_1,T,TC, and Q have been calculated using (7.2)for solution of the system of non linear equation using Newton Raphson method with the help of Mat lab.

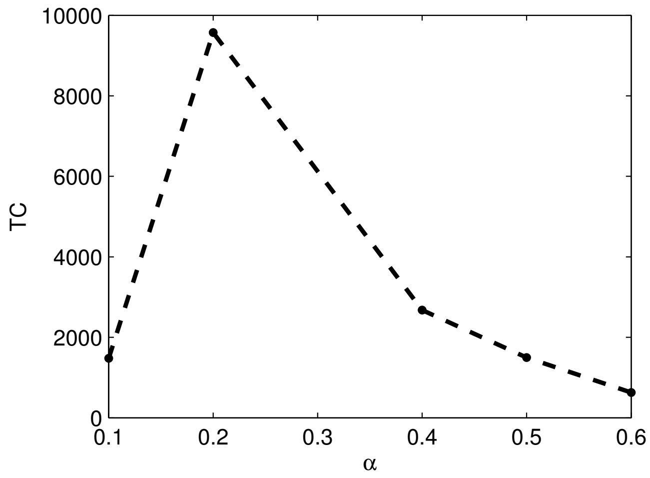

FIGURE 4. Variation of optimal cost TC with respect to deterioration parameter α

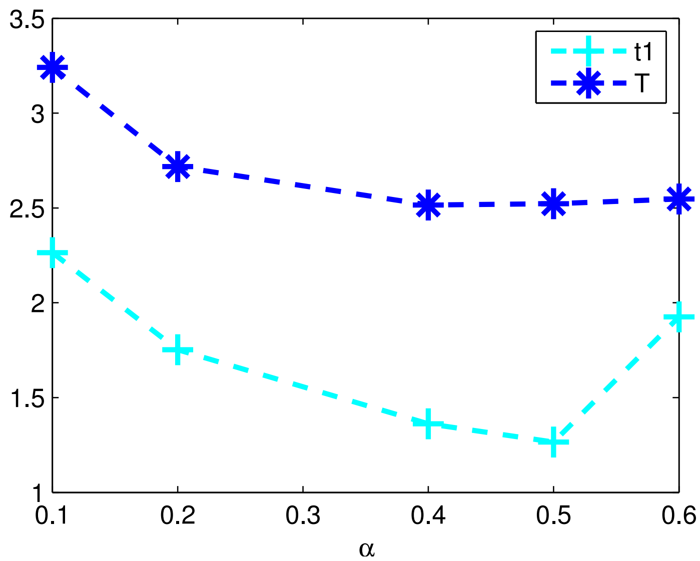

FIGURE 5. Variation of optimal time t_1 and T with respect to deterioration parameter α

EXAMPLE A. Consider an inventory system with parameters as

$$A = 8, \quad Cd = 1, \quad Ch = 5, \quad Cs = 1, \quad a = 100, \quad b = 3,$$

$$x = 0.0023, \quad y = 0.09, \quad t = 2, \quad \beta = 2, \quad T = 1.47,$$

$$\pi = 12, \quad \alpha = 0.13, \quad t_1 = 1.42, \quad C = 4.$$

EXAMPLE B. We consider an inventory system which verifies the assumptions described above with the parameters as

$$A = 8, \quad Cd = 1, \quad Ch = 5, \quad Cs = 1, \quad a = 100, \quad b = 3,$$

TABLE 1. Numerical Example (a)

α	t_1	T	TC	Q
0.05	1.43409	1.48441	1016.89	335.2650
0.09	1.42910	1.48114	1027.032	338.0354
0.13	1.42424	1.47801	1037.01	340.7916
0.17	1.41950	1.47501	1046.85	343.5326
0.21	1.41489	1.47214	1056.53	346.2593
0.25	1.41039	1.46940	1066.10	348.9815
0.30	1.40492	1.46615	1079.72	352.7679
0.40	1.39448	1.46019	1100.97	359.1348

$$x = 0.0023, \quad y = 0.09, \quad t = 2, \quad \beta = \frac{1}{2}, \quad T = 1.47,$$
$$\pi = 12, \quad \alpha = 0.13, \quad t_1 = 1.42, \quad C = 5.$$

TABLE 2. Numerical Example (b)

α	t_1	T	TC	Q
0.10	2.264520	3.240990	1479.2	8874
0.20	1.504149	2.553645	9574.46	6346
0.40	1.362177	2.515763	2677.7	5003.6
0.50	1.266038	2.522837	1498.2	5250.3
0.60	1.92582	2.547586	627.1629	5640.1

8.1. **OBSERVATIONS.** On the basis of numerical experiment performed, it has been observed from table 1 for case $\beta = 2$, higher the deterioration parameter or lesser the fresh life time of product optimal cost of the system increases but the optimal value of T and t_1 decreases. This reflects that as deterioration rate increases shortage will come faster which leads to reduction in holding cost but rise in deterioration cost dominates the holding cost of the system. Furthermore as the deterioration parameter α increases the optimal order quantity Q also increases consistently.

On the other hand it is found from table 2 for the case $\beta = \frac{1}{2}$, as the value of the optimal value T decrease up to $\alpha = 0.50$ and the value of t_1 decreases constantly.

This shows that $\alpha = 20$ is optimal point where the total cost per unit time is minimized and corresponding optimal order quantity is the EOQ which would have to order for the next replenishment.

9. CONCLUSION

This paper presented a mathematical model of an inventory system in which demand depending upon stock level and time with various value of β. It gives more flexibility of the demand pattern and more general tot he study done so far with the condition to minimize the total cost of the system.

This paper also adopted a new method in order to solve the differential (5.4) and (5.5) as the solution 6.1 and 6.2 in terms of series. From these solutions we known that the stock level of the product usually has a positive impact on its demand of the product. It has been detected that the failure rate and life anticipation of many item can be expressed in terms of Weibull distribution.

The model is helpful for taking the decision in the sense that to determine the appropriate time in order to avoid the shortage so that the business can be run smoothly without facing the problem of shortage.

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