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ORDERING COST DEPENDENT LEAD TIME IN INTEGRATED INVENTORY MODEL

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ABSTRACT: This paper deals with an integrated vendor buyer supply chain and investigates the impact of lead time reduction on the modified continuous review inventory system where lead time and ordering cost reductions act dependently. Two models are constructed based on the probability distribution of the lead time demand. The objective of this paper is to minimize the total system cost by simultaneously optimizing the order quantity, lead time and reorder point. Moreover, the lead time demand is assumed to be normally distributed. A procedure of finding the optimal solution is developed. Furthermore, the sensitivity analysis is included and the numerical examples are given to illustrate the results. Finally, the graphical representation is presented to illustrate the model.

AMS Subject Classification: 90B05

1. INTRODUCTION

In the dynamic, competitive environment, supply chain management has emerged as a popular production and logistics strategy for many contemporary firms, and the just-in-time (JIT) purchasing plays a crucial role in such supply chain environments. Companies are using JIT purchasing to gain and maintain a competitive advantage.

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The benefits of JIT purchasing include small lot sizes, frequent deliveries, consistent high quality, reduction in lead times, decrease in inventory levels, lower setup cost and ordering cost, and close supplier ties. In recent years, companies have found that there are substantial benefits from establishing a long-term sole-supplier relationship with supplier [15]. In the JIT environment, a close cooperation exists between supplier and purchaser to solve problems together, and thus maintains stable, long-term relationships. In the production environment, lead time plays an important role today's logistics management. Defined as the time that elapses between the placements of an order into inventory [25], lead time may influence customer service and impact inventory costs. As the Japanese example of just-in-time-production has shown, consequently reducing lead time may increase productivity and improve the competitive position of the company [27]. Although lead time can be constant or variable, it is often treated as a prescribed parameter in most of the inventory management literature. Therefore, the lead time crashing cost function is a piecewise linear function [14] [18] [20]. The number of advantages and benefits has been associated in the efforts of control of the lead time (which is a goal of JIT inventory management philosophies that emphasizes high quality and keeps low inventory level and lead time to a practical minimum). Lead time management is a significant issue in production and operation management. In many practical situations lead time can be reduced using an added crashing cost. In other words, lead time is controllable. The crashing of lead time mainly consists of the following components: order preparation, order transit, supplier lead time and delivery time [26].

The controllable lead time becomes a prominent issue and its control leads to many benefits. Ouyang developed lead time reduction inventory models under various crashing cost function and practical situations. The integrated inventory management system is a common practice in the global markets and provides economic advantages for both the vendor and the buyer. In recent years, most integrated inventory management systems have focused on the integration between vendor and buyer. Once they form a strategic alliance in order to minimize their own cost or maximize their own profit, then trading parties can collaborate and share information to achieve improved benefits. Therefore, several authors ([1], [3], [5], [6], [12], [19], [21], [29], [28], [30], [31]) have presented the integrated inventory management system. [22] developed a investing in setups in the EOQ model.

In recent years, many studies have focused on the benefit from ordering cost reduction in the inventory systems but only from the single party's viewpoint ([22], [4], [13], [8]). However, considering the dyadic relationship between the vendor and buyer is necessary for implementing an EDI-based ordering system since the implementation needs both the trading partners to interchange transaction documents, to standardize transaction procedures, and to integrate related applications [10]. To address the vendor buyer integration of EDI, Banerjee and Banerjee [2] considered an EDI-based vendor-managed inventory VMI system in which the vendor makes all replenishment decisions for his/her buyers to improve the joint inventory cost. Their work focused solely on the inventory policy by assuming that an EDI system has already been operated between vendor and buyers and hence no ordering cost will incur for both parties.

All the aforementioned integrated vendor-buyer inventory systems treat the ordering cost and/or lead time as constants. However, in the practical market, ordering cost and lead time can be controlled and reduced in various ways. For example, lead time can be reduced at an added crashing cost; ordering cost reduction can be attained through worker training, procedural changes, and specialized equipment acquisition; in other words, the lead time is controllable, and the ordering cost can be reduced through further investment. It has been a trend by shortening the lead time and reducing ordering cost; we can lower the safety stock, reduce the stockout loss, and improve the service level to the customer. In this paper, [19] proposed continuous review inventory model to study the effects of lead time and ordering cost reductions. We note that the lead time and ordering cost reductions in [19] models are assumed to act independently; however, this is only one of the possible cases. In practice, the lead time may accompany the reduction of ordering cost, and vice versa. For example, according to [24], the implementation of electronic data interchange (EDI) may reduce the lead time and ordering cost simultaneously. Therefore it is more reasonable to assume that ordering cost reduction vary according to different lead times. And then, their functional relationship may be as linear, logarithmic, exponential and the likes.[10] developed a critical perspective on information technology management in the case of electronic data interchange. [11] proposed an integrated inventory model for a single supplier and single customer problem. [17] developed a setup cost and lead time reductions in lot size reorder point models with an imperfect production process and [16] considered a lead time and distributional assumptions in continuous review inventory models.

In this paper is to investigate the effect of lead time reduction on a continuous review inventory system. Two models are constructed based on the probability distribution of the lead time demand. In the first model, we assume that the lead time demand to be normal distribution. In the second model, we consider the distribution free approach for the lead time demand. For the second model, only mean and standard deviation is known. And we consider the case where the lead time and ordering cost reductions with linear function, then consider the logarithmic functional relationship. The aim is to minimize the total system cost by simultaneously optimizing the order quantity, lead time and reorder point. An algorithm is developed to optimize the joint expected total cost for the buyer and the vendor. An algorithm is also implemented for the distribution free approach. Some numerical examples are given to illustrate our model.

The remainder of this paper is organized as follows. In Section 2 we provide the notations and assumptions. The mathematical model is developed in Section 3. We formulate a single-vendor single-buyer inventory model in Section 4. Subsections 4.1 and 4.2 describe the solution procedure. An efficient algorithm is developed to obtain the optimal solution in Section 5. For both models we present numerical examples and sensitivity analysis. In Section 6 marginal implication are discussed. And finally in Section 7 the conclusion of this study is summarized.

2. NOTATIONS AND ASSUMPTIONS

To develop the mathematical model, let us introduce the following notations and assumptions.

2.1. NOTATIONS

The following notations will be used to develop our model.

D	The demand rate in units per unit time
P	The production rate per unit time
n	The number of shipments from the vendor to the buyer
Q	The quantity ordered by the buyer
R	The reorder point
S	The setup cost of the vendor per year
A	The ordering cost of the buyer per order, $0 \leq A \leq A_0$
h_v	The holding cost rate of the vendor per unit per unit time
h_b	The holding cost rate of the buyer per unit per unit time
C_v	The unit production cost paid by the vendor
C_b	The unit purchase cost paid by the vendor
π	The unit backlogging cost for the buyer
A_0	The initial ordering cost of the buyer
X	The lead time demand
L	The length of the lead time for the buyer
$E(.)$	The mathematical expectation

x^+	The maximum value of x and 0
$f_x(X)$	The probability density function (pdf) of X with finite mean DL and standard deviation $\sigma\sqrt{L}$
σ	The standard deviation of demand per unit time

2.2. ASSUMPTIONS

1. An integrated vendor-buyer model is considered.

2. The buyer use the continuous review policy for all products and the economic order quantity Q, the vendor manufactures the lot nQ with finite production rate P and the buyer places an order when the level of inventory reaches the reorder point R.

3. The reorder point r=expected demand during lead time + safety stock (SS), and SS=k. (standard deviation of lead time demand), i.e.,
$$R = DL + k\sigma\sqrt{L},$$
where k is the safety factor.

4. Shortages are allowed and fully backordered.

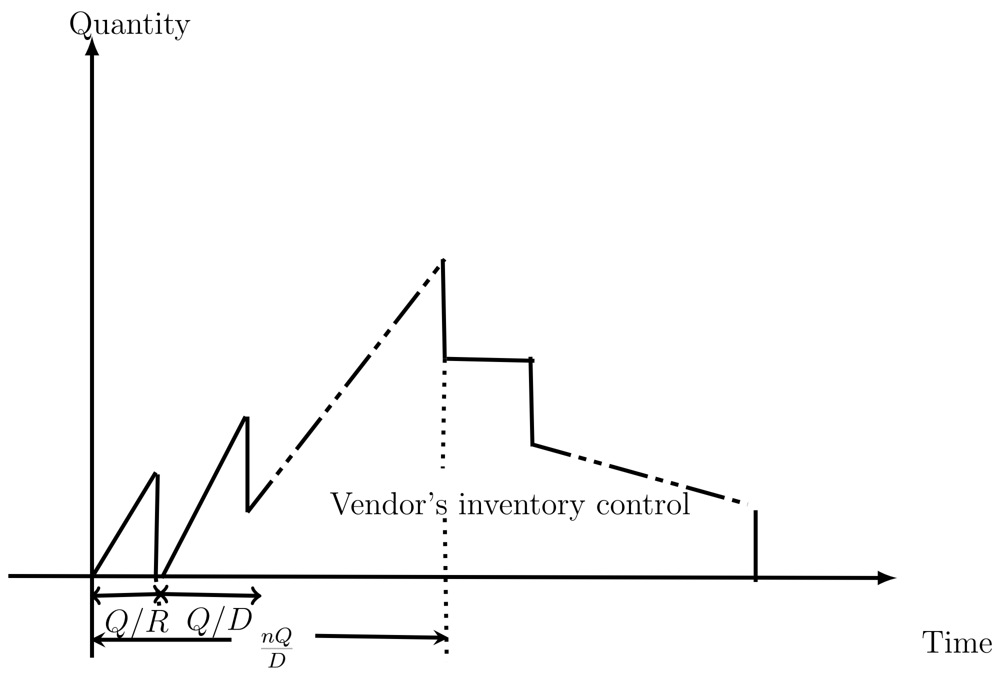
5. For all products, the lead time L consists of n mutually independent components. The i^{th} component has a normal duration b_i, minimum duration a_i, and crashing cost per unit time c_i such that $c_1 \leq c_2 \leq \cdots \leq c_n$. The components of lead time are crashed one at a time starting from the first component because it has the minimum unit crashing cost, and then the second component, and so on. Let $L_0 = \sum_{i=1}^{m} b_i$, and L_i be the length of the lead time with components $1, 2, \ldots, i$ crashed to their minimum duration, then L_i can be expressed as $L_i = L_0 - \sum_{j=1}^{i}(b_j - a_j)$, $i = 1, 2, \ldots, m$; and for all products, the lead time crashing cost per cycle $C(L)$ is given by
$$C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j), \quad L \in [L_i, L_{i-1}].$$

6. The reduction of lead time L accompanies a decrease of ordering cost A and A_0 is a strictly concave function of L, i.e., $A'(L) > 0$ and $A"(L) < 0$.

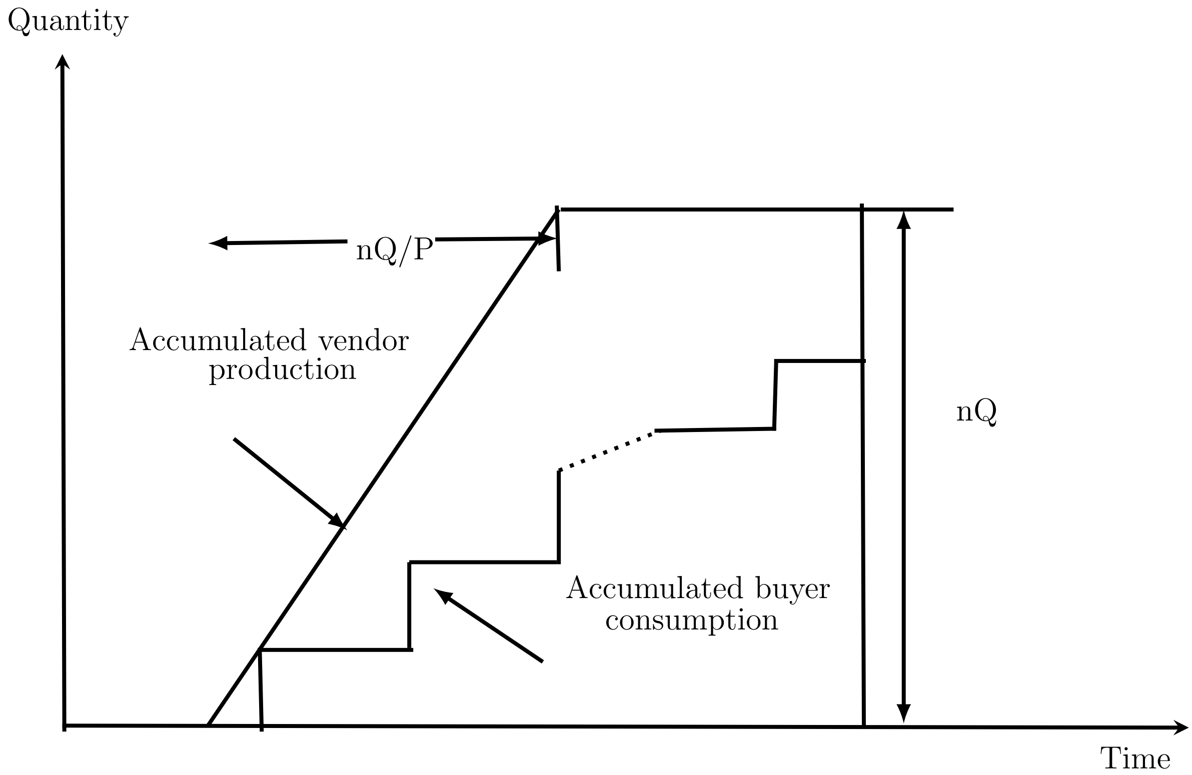
7. The transportation cost per unit time from the vendor to the buyer is constant and independent of the quantity ordered. Thus, the total transportation cost per unit time is neglected.

3. MODEL DEVELOPMENT

In this paper, we restrict our attention to an integrated inventory model consisting of a single vendor who provides one type of product to a single buyer. The integrated inventory model is designed as follows. If the buyer orders quantity Q, then the vendor produces nQ at one set-up, with a finite production rate R, and $R > D$, in order to reduce its set-up cost, and then for reducing the inventory cost, the vendor as soon as delivers the lot size Q to the buyer over n times, where n is a positive integer. Therefore, the length of each production cycle for the vendor is nQ/D and the length of each ordering cycle for the buyer is Q/D.



The ordering cost is AD/Q. On the other hand, we assume that the integrated production inventory model allows shortages with partial backorder. Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point r. That is, a new order Q is placed whenever the reorder point is reached. The expected inventory level before receipt of an order is $R - DL$ and the expected inventory level immediately after the delivery of quantity Q is $Q+(R-DL)$. Hence, the average inventory over a cycle can be written as $\frac{Q}{2}+R-DL$ which implies that the buyer's expected holding cost per unit time becomes $r_b c_b[\frac{Q}{2} + R - DL]$.



Inventory pattern of the vendor and buyer

The lead time demand follows a normal distribution, so the lead time demand X has a pdf $f_x(X)$ with finite mean DL and standard deviation $\sigma\sqrt{L}$ The reorder point r=expected demand during lead time + safety stock (SS), and SS=k. (standard deviation of lead time demand), i.e., R=DL+$k\sigma\sqrt{L}$, where k is the safety factor. Shortages occur when $X > R$, then the expected demand shortage at the end of the cycle is given by $E(X-r)^+ = \int_R^\infty (x-R)dF(x)$. The expected shortages cost per unit time is $\frac{\pi D E(X-r)^+}{Q}$. The lead time crashing cost per unit is $\frac{DC(L)}{Q}$.

Therefore, the total expected annual cost of the buyer is

$$ETC_b(Q, R, L) = \frac{AD}{Q} + r_b C_b(\frac{Q}{2} + R - DL) + \frac{\pi D}{Q} E(X-R)^+ + \frac{DC(L)}{Q}. \quad (1)$$

For each production run, the set-up cost per unit time is SD/nQ. When the vendor produces the first Q units, he will deliver them to the buyer, after that the vendor will make the deliver on the average every Q/D unit of time until the inventory level falls to zero. Therefore, the total expected cost of the vendor per unit time is

$$ETC_v(Q, n) = \frac{SD}{nQ} + r_v C_v \frac{Q}{2}[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}]. \quad (2)$$

We assume that demand during lead time is normally distributed with mean $DL(Q)$ and standard deviation $\sigma\sqrt{L}$. In this case, $S = k\sigma\sqrt{L}$, and the expected shortage at the end of the cycle is

$$E(X-R)^+ = \int_R^\infty (x-R)dF(x) = \sigma\sqrt{L}\psi(k)$$

where $\psi(k) = \phi(k) - k[1-\Phi(k)]$, ϕ=standard normal probability density function, and Φ=cumulative distribution function of the normal distribution. The safety factor k is assumed as a decision variable instead of R.

Consequently, the joint expected total cost per unit time for the vendor and the buyer can be expressed as

$$\begin{aligned} JETC(Q,k,A,L,n) &= ETC_b(Q,k,L) + ETC_v(Q,n) \\ &= \frac{AD}{Q} + r_bC_b(\frac{Q}{2} + R - DL) + \frac{\pi D}{Q}E(X-R)^+ + \frac{DC(L)}{Q} + \frac{SD}{nQ} \\ &\quad + r_vC_v\frac{Q}{2}\left[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}\right]. \end{aligned} \quad (3)$$

3.1. LINEAR FUNCTION

In this subsection, we assume that lead time and ordering cost reductions act dependently with the following relationship [7]

$$\frac{L_0 - L}{L_0} = \alpha\frac{A_0 - A}{A_0} \quad (4)$$

where $\alpha(>0)$ is a constant scaling parameter to describe the linear relationship between percentages of reductions in lead time and ordering cost. [7] utilized relationship (4) to formulate the inventory problems by treating Q and L as decision variables. In this paper also Q and L are decision variables and in addition to these variables, k is considered to be a decision variable.

By considering relationship (3), the ordering cost A can be written as a linear function of L, that is given by,

$$A(L) = u + vL \quad (5)$$

where $u = (1-\frac{1}{\alpha})A_0$ and $v = \frac{A_0}{\alpha L_0}$.

Substituting (5) in (3) we receive

$$\begin{aligned} \min JETC(Q,k,L,n) &= \frac{D}{Q}[(u+vL) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)] \\ &\quad + \frac{Q}{2}[r_bC_b + r_vC_v[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}]] + r_bC_bk\sigma\sqrt{L}. \end{aligned} \quad (6)$$

In order to solve this nonlinear program, and for a fixed positive integer n, we take the partial derivatives of $JETC(Q, k, L, n)$ with respect to Q, k and L to obtain the optimal solution.

$$\begin{aligned}
\frac{\partial JETC(Q,k,L,n)}{\partial Q} &= -\frac{D}{Q^2}[(u+vL)+\frac{S}{n}+\pi\sigma\sqrt{L}\psi(k)+C(L)] \\
&\quad +\frac{1}{2}[r_bC_b+r_vC_v[n(1-\frac{D}{P})-1+\frac{2D}{P}]], \qquad (7)\\
\frac{\partial JETC(Q,k,L,n)}{\partial k} &= \frac{D}{Q}\pi\sigma\sqrt{L}[\Phi(k)-1]+r_bC_b\sigma\sqrt{L}, \qquad (8)\\
\frac{\partial JETC(Q,k,L,n)}{\partial L} &= \frac{D}{Q}v+\frac{D}{2Q\sqrt{L}}\pi\sigma\psi(k)+\frac{D}{Q}\frac{\partial C(L)}{\partial L}+\frac{r_bC_bk\sigma}{2\sqrt{L}}. \qquad (9)
\end{aligned}$$

By examining the second order sufficient conditions ($SOSC$), it can be verified that $JETC(Q, k, L, n)$ is concave in L.

$$i.e., \frac{\partial^2 JETC(Q,k,L,n)}{\partial L^2} = -\frac{D}{4Q}\pi\sigma\psi(k)L^{-3/2}-\frac{1}{4}r_bC_bk\sigma L^{-3/2}<0.$$

Hence for fixed Q, k and n, the minimum joint expected total cost is attained at the end points of the interval $[L_i, L_{i-1}]$. Now for fixed positive integer n, the values of Q and $\Phi(k)$ are obtained by equating the equations (7) and (8) to zero as given by

$$\begin{aligned}
Q &= \left\{\frac{2D[(u+vL)+\frac{S}{n}+\pi\sigma\sqrt{L}\psi(k)+C(L)]}{r_bC_b+r_vC_v[n(1-\frac{D}{P})-1+\frac{2D}{P}]}\right\}^{\frac{1}{2}}, \qquad (10)\\
\Phi(k) &= 1-\frac{r_bC_bQ}{D\pi}. \qquad (11)
\end{aligned}$$

Theoretically, for fixed $L = [L_i, L_{i-1}]$, from the above equations (10) and (11), we can get the values of Q and k. Moreover, it can be shown that the SOSC are satisfied since the Hessian matrix is positive definite at point (Q, k) (see the appendix for the proof).

Proposition 1. *For a fixed Q, k and L, JETC(Q,k,L,n) is convex in n.*

Proof. Taking the first and second partial derivatives of JETC(Q,k,L,n) in (6) with respect to n, we have

$$\begin{aligned}
\frac{\partial JETC}{\partial n} &= -\left(\frac{SD}{Qn^2}\right)+\frac{r_vC_vQ}{2}\left(1-\frac{D}{P}\right),\\
\frac{\partial^2 JETC}{\partial n} &= \frac{2SD}{Qn^3}>0.
\end{aligned}$$

Therefore, for fixed Q, k and L, JETC(Q,k,L,n) is convex in n.

3.2. LOGARITHMIC FUNCTION

In this subsection, we assume that the lead time and ordering cost reductions act dependently with the following relationship:

$$\frac{A_0 - A}{A_0} = \tau \ln(\frac{L}{L_0}), \tag{12}$$

where $\tau(< 0)$ is a constant scaling parameter to describe the logarithmic relationship between percentages of reductions in lead time and ordering cost. In this case, the ordering cost A can be written as

$$A(L) = d + e \ln L, \tag{13}$$

where $d = A_0 + \tau A_0 \ln L_0$ and $e = -\tau A_0 > 0$.

Substituting (13) in (3), we get

$$\begin{aligned} \min JETC(Q,k,L,n) = {} & \frac{D}{Q}[(d + e\ln L) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)] \\ & + \frac{Q}{2}[r_bC_b + r_vC_v[n(1 - \frac{D}{P}) - 1 + \frac{2D}{P}]] + r_bC_bk\sigma\sqrt{L}. \end{aligned} \tag{14}$$

Again, the approach employed in the Subsection 3.1 is utilized to solve (14), i.e.,

$$\begin{aligned} \frac{\partial JETC(Q,k,L,n)}{\partial Q} &= -\frac{D}{Q^2}[(d + e\ln L) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)] \\ &\quad + \frac{1}{2}[r_bC_b + r_vC_v[n(1 - \frac{D}{P}) - 1 + \frac{2D}{P}]], \qquad (15) \\ \frac{\partial JETC(Q,k,L,n)}{\partial k} &= \frac{D}{Q}\pi\sigma\sqrt{L}[\Phi(k) - 1] + r_bC_b\sigma\sqrt{L}, \qquad (16) \\ \frac{\partial JETC(Q,k,L,n)}{\partial L} &= \frac{De}{QL} + \frac{D}{2Q\sqrt{L}}\pi\sigma\psi(k) + \frac{D}{Q}\frac{\partial C(L)}{\partial L} + \frac{r_bC_bk\sigma}{2\sqrt{L}}. \qquad (17) \end{aligned}$$

It can be shown that, for fixed (Q, k), JETC(Q,k,L,n) is concave in $L \in [L_i, L_{i-1}]$, because

$$\frac{\partial^2 JETC(Q,k,L,n)}{\partial L^2} = -\frac{De}{L^2Q} - \frac{D}{4Q}\pi\sigma\psi(k)L^{-3/2} - \frac{1}{4}r_bC_bk\sigma L^{-3/2} < 0.$$

Hence for fixed Q, k and n, the minimum joint expected total cost is attained at the end points of the interval $[L_i, L_{i-1}]$. Now for fixed positive integer n, the values of Q and $\Phi(k)$ are obtained by equating the equations (15) and (16) to zero as given by

$$Q = \left\{ \frac{2D[(d + e\ln L) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)]}{r_bC_b + r_vC_v[n(1 - \frac{D}{P}) - 1 + \frac{2D}{P}]} \right\}^{\frac{1}{2}}, \tag{18}$$

$$\Phi(k) = 1 - \frac{r_b C_b Q}{D\pi}. \tag{19}$$

For fixed $L = [L_i, L_{i-1}]$, from the above equations (18) and (19), we can get the values of Q and k. Moreover, it can be shown that the SOSC are satisfied since the Hessian matrix is positive definite at point (Q,k) (the proof is similar to that given in the appendix).

Proposition 2. *For a fixed Q, k and L, JETC(Q,k,L,n) is convex in n.*

Proof. Taking the first and second partial derivatives of JETC(Q,k,L,n) in (14) with respect to n, we have

$$\begin{aligned}\frac{\partial JETC}{\partial n} &= -\left(\frac{SD}{Qn^2}\right) + \frac{r_v C_v Q}{2}\left(1 - \frac{D}{P}\right),\\ \frac{\partial^2 JETC}{\partial n} &= \frac{2SD}{Qn^3} > 0.\end{aligned}$$

Therefore, for fixed Q, k and L, JETC(Q,k,L,n) is convex in n.

Solution Algorithm for Linear Case (Normal Distribution)

Step 1 Set $n = 1$.

Step 2 For each L_i, $i = 0, 1, 2, ..., m$ preform step (2.1) and (2.5).

Step 2.1 Set $k_{i1} = 0$ (implies $\psi(k_{i1}) = 0.3989$).

Step 2.2 Substitute $\psi(k_{i1})$ into equation (10) and find Q_{i1}.

Step 2.3 Using Q_{i1} to evaluate the value of $\Phi(k_{i2})$ from the equation (11).

Step 2.4 For the values of $\Phi(k_{i2})$, find the value of k_{i2} from the normal table and hence to find $\psi(k_{i2})$.

Step 2.5 Repeat (2.1) to (2.4) until no changes occurs in the values of Q_i and k_i. We denote this values by Q_i^* and k_i^*.

Step 3 Find $\min\limits_{i=0,1,2,\dots,m} JETC(Q_i^*, k_i^*, L_i, n)$.

Step 4 Evaluate $JETC(Q_i^*, k_i^*, L_i, n)$. If $JETC(Q_i^*, k_i^*, L_i, n) = \min\limits_{i=0,1,2,\dots,m} JETC(Q_i^*, k_i^*, L_i, n)$, then $JETC(Q_i^*, k_i^*, L_i, n)$ is the optimal solution.

Step 5 Set $n = n + 1$ and if $JETC(Q_n^*, k_n^*, L_n, n) \leq JETC(Q_n^*, k_n^*, L_n, n-1)$ repeat the steps 2 to 4. Otherwise go to step 6. to get $JETC(Q_{(n)}^*, k_{(n)}^* L_{(n)}, n)$.

Step 6 Set $JETC(Q_{(n)}^*, k_{(n)}^*, L_{(n)}, n) = JETC(Q_{(n-1)}^*, k_{(n)}^*, L_{(n-1)}, n-1)$. Then (Q^*, k^*, L, n) is the optimal solution.

Once we obtain the (Q^*, k^*, L, n), the reorder point is $R^* = DL + k\sigma\sqrt{L}$. And the optimal ordering cost is $A^* = u + vL$ (for linear case).

Solution algorithm for logarithmic case (Normal distribution)

Step 1 Set $n = 1$.

Step 2 For each L_i, $i = 0, 1, 2, ..., m$ preform step (2.1) and (2.5).
Step 2.1 Set $k_{i1} = 0$ (implies $\psi(k_{i1}) = 0.3989$).
Step 2.2 Substitute $\psi(k_{i1})$ into equation (18) and find Q_{i1}.
Step 2.3 Using Q_{i1} to evaluate the value of $\Phi(k_{i2})$ from the equation (19).
Step 2.4 For the values of $\Phi(k_{i2})$, find the value of k_{i2} from the normal table and hence to find $\psi(k_{i2})$.
Step 2.5 Repeat (2.1) to (2.4) until no changes occurs in the values of Q_i and k_i. We denote this values by Q_i^* and k_i^*.

Step 3 Find $\min_{i=0,1,2,...,m} JETC(Q_i^*, k_i^*, L_i, n)$.

Step 4 Evaluate $JETC(Q_i^*, k_i^*, L_i, n)$. If $JETC(Q_i^*, k_i^*, L_i, n) = \min_{i=0,1,2,...,m} JETC(Q_i^*, k_i^*, L_i, n)$, then $JETC(Q_i^*, k_i^*, L_i, n)$ is the optimal solution.

Step 5 Set $n = n + 1$ and if $JETC(Q_n^*, k_n^*, L_n, n) \leq JETC(Q_n^*, k_n^*, L_n, n-1)$ repeat the steps 2 to 4. Otherwise go to step 6. to get $JETC(Q_{(n)}^*, k_{(n)}^* L_{(n)}, n)$.

Step 6 Set $JETC(Q_{(n)}^*, k_{(n)}^*, L_{(n)}, n) = JETC(Q_{(n-1)}^*, k_{(n)}^*, L_{(n-1)}, n-1)$. Then (Q^*, k^*, L, n^*) is the optimal solution.

Once we obtain the (Q^*, k^*, L, n^*), the reorder point is $R^* = DL^* + k\sigma\sqrt{L}$. And the optimal ordering cost is $A^* = d + e\ln L$.

4. DISTRIBUTION FREE APPROACH

We consider the distribution free approach for the same model mentioned above. The information about the probability distribution of the lead time demand is often quiet limited. we only assume that the density function F of lead time demand X belongs to the class of density functions with a known finite mean DL and standard deviation $\sigma\sqrt{L}$. Only mean and standard deviation are known. The value of $E(X-R)^+$ cannot be determined exactly as the probability distribution of the lead time demand X is unknown. we try to use a minimax distribution free procedure to solve this problem. i.e., our problem is to solve

$$\min_{Q,k,L,n} \max_{F\in\Omega} JETC(Q,k,L,n). \tag{20}$$

The following proposition is used to approximate the value of $E(X-R)^{+}$ which was asserted by [9].

Proposition 3. *For any $F \in \Omega$, the following inequality always holds for*

$$E(X-R)^{+} \leq \frac{1}{2}\sqrt{[\sigma^2 L + (R-DL)^2} - (R-DL)]. \tag{21}$$

4.1. DISTRIBUTION FREE MODEL FOR LINEAR FUNCTION

Here, $R{=}DL{+}k\sigma\sqrt{L}$ as mentioned previously, and for any probability distribution of the lead time demand X, the above inequality (21) always holds. By using relation (6) and the inequality (21), model (20) can be written as

$$\begin{aligned} minJETC_f(Q,k,L,n) &= \frac{D}{Q}[(u+vL) + \frac{S}{n} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2}-k) + C(L)] \\ &+ \frac{Q}{2}[r_bC_b + r_vC_v[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}]] + r_bC_bk\sigma\sqrt{L}. \quad (22) \end{aligned}$$

Here f denotes the distribution free case.

By analogous arguments in the normal distribution demand case, we can show that $minJETC_f(Q,k,L,n)$ is a concave function of $L \in [L_i, L_{i-1}]$ for fixed (Q,k,n). Thus, for fixed Q and k, the minimum value of $minJETC_f(Q,k,L,n)$ will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for a given value of $L \in [L_i, L_{i-1}]$, $minJETC_f(Q,k,L,n)$ is convex in Q and k. Hence for fixed $L \in [L_i, L_{i-1}]$, the minimum value of (22) will occur at the point Q and k which satisfies $\frac{\partial JETC_f(Q,k,L,n)}{\partial Q} = 0$, $\frac{\partial JETC_f(Q,k,L,n)}{\partial k} = 0$ simultaneously. Therefore, the resulting solutions are

$$Q = \left\{\frac{2D[(u+vL) + \frac{S}{n} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2}) + C(L)]}{r_bC_b + r_vC_v[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}]}\right\}^{\frac{1}{2}}, \tag{23}$$

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{r_bC_bQ}{D\pi}. \tag{24}$$

4.2. DISTRIBUTION FREE MODEL FOR LOGARITMIC FUNCTION

Here, $R{=}DL{+}k\sigma\sqrt{L}$ as mentioned previously, and for any probability distribution of the lead time demand X, the above inequality (21) always holds. By using relation

(14) and the inequality (21), model (20) can be written as

$$minJETC_f(Q,k,L,n) = \frac{D}{Q}[(D + e\ln L) + \frac{S}{n} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2} - k) + C(L)] \\ + \frac{Q}{2}[r_bC_b + r_vC_v[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}]] + r_bC_bk\sigma\sqrt{L}. \quad (25)$$

Here f denotes the distribution free case.

By analogous arguments in the normal distribution demand case, We can show that $minJETC_f(Q,k,L,n)$ is a concave function of $L \in [L_i, L_{i-1}]$ for fixed (Q,k,n). Thus, for fixed Q and k, the minimum value of $minJETC_f(Q,k,L,n)$ will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for a given value of $L \in [L_i, L_{i-1}]$, $minJETC_f(Q,k,L,n)$ is convex in Q and k. Hence for fixed $L \in [L_i, L_{i-1}]$, the minimum value of (25) will occur at the point Q and k which satisfies $\frac{\partial JETC_f(Q,k,L,n)}{\partial Q} = 0$, $\frac{\partial JETC_f(Q,k,L,n)}{\partial k} = 0$ simultaneously. Therefore, the resulting solutions are

$$Q = \left\{\frac{2D[(u+vL) + \frac{S}{n} + \frac{1}{2}\pi\sigma\sqrt{L}(\sqrt{1+k^2}) + C(L)]}{r_bC_b + r_vC_v[n(1-\frac{D}{P}) - 1 + \frac{2D}{P}]}\right\}^{\frac{1}{2}}, \quad (26)$$

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{r_bC_bQ}{D\pi}. \quad (27)$$

Solution algorithm for linear case (Free distribution)

Step 1 Set $n = 1$.

Step 2 For each L_i, $i = 0,1,2,...,m$ preform step (2.1) and (2.5).

Step 2.1 Set $k_{i1} = 0$ (implies $\psi(k_{i1}) = 0.3989$).

Step 2.2 Substitute $\psi(k_{i1})$ into equation (23) and find Q_{i1}.

Step 2.3 Using Q_{i1} to evaluate the value of k_{i2} from the equation (24).

Step 2.4 Repeat (2.1) to (2.3) until no changes occurs in the values of Q_i and k_i.

We denote this values by Q_i^{**} and k_i^{**}.

Step 3 Find $\min_{i=0,1,2,...,m} JETC(Q_i^{**}, k_i^{**}, L_i, n)$.

Step 4 Set $n = n+1$ and if $JETC(Q_n^{**}, k_n^{**}, L_n, n)$ $\leq JETC(Q_n^{**}, k_n^{**}, L_n, n-1)$ repeat the steps 2 to 4.

Otherwise go to step 6. to get $JETC(Q_{(n)}^{**}, k_{(n)}^{**}, L_{(n)}, n)$.

Step 5 Set $JETC(Q_{(n)}^{**}, k_{(n)}^{**}, L_{(n)}, n) = JETC(Q_{(n-1)}^{**}, k_{(n)}^{**}, L_{(n-1)}, n-1)$.

Then (Q^{**}, k^{**}, L, n) is the optimal solution.

Once we obtain the (Q^{**}, k^{**}, L, n), the reorder point is $R^{**} = DL + k\sigma\sqrt{L}$. And the optimal ordering cost is $A^{**} = u + vL$.

Utilizing a similar procedure as proposed in the above algorithm can be performed to find the optimal solution in logarithmic case (Free distribution), in which the optimal values of order quantity, safety factor and lead time, respectively are denoted by (Q^{**}, k^{**}, L, n) and reorder point is denoted by $R^{**} = DL + k\sigma\sqrt{L}$.

5. NUMERICAL EXAMPLES

The optimality of the proposed algorithms is illustrated through numerical examples. In this section, numerical analysis is conducted. The solutions to this example is obtained by using the computer MatLab software R2008a. The computational effort and time are small for the proposed algorithm and it is simple to implement. Let us consider the continuous review inventory system with the following data used in [23]: D=600 units/year, C_b=\$100/unit, π=\$50/unit, σ=7 units/week, p=2000 units/year, C_v=\$70/unit, r_b=\$0.2/unit/year, r_v=\$0.2/unit/year, A_0=200, and the lead time has three components with data shown in Table 1 and summarized lead time are discussed in Table 2.

Table 1: Lead time component with data.

Lead time component i	Normal duration b_i (days)	Minimum duration cost a_i (days)	Unit crashing c_i (\$/days)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 2: Summarized lead time data.

Lead time (week)	$C(L)$
8	0
6	5.6
4	22.4
3	57.4

Example 1. Suppose that the lead time demand follows a normal distribution and consider the case that the relationship between lead time and ordering cost is linear. We solve the case when α= 1.25. Applying the algorithm in linear case (Normal distribution), the results of the solution procedures are summarized in Table 3 and

the corresponding curve of the minimum joint expected total cost is plotted in figure 1.

Table 3: The optimal solutions for linear relationship case using normal distribution (L_i in weeks)

α	i	L_i	n	A_i^*	Q_i^*	k_i^*	R_i^*	$JETC$
	0	8	3	200	143	1.30	118	6781
1.25	1	6	3	160	139	1.32	92	6535
	2	4	3	120	136	1.33	65	6321
	3	3	3	100	137	1.33	51	6317

Example 2. Suppose that the lead time demand follows a normal distribution and consider the case that the relationship between lead time and ordering cost is logarithmic. We solve the case when τ= -0.5. Applying the algorithm in logarithmic case (Normal distribution), the results of the solution procedures are shown in Table 4 and the corresponding curve of the minimum joint expected total cost is plotted in figure 2.

Table 4: The optimal solutions for logarithmic relationship case using normal distribution (L_i in weeks)

τ	i	L_i	n	A_i^*	Q_i^*	k_i^*	R_i^*	$JETC$
	0	8	3	200	143	1.30	118	6781
-0.5	1	6	3	171	111	1.32	92	6579
	2	4	3	131	98	1.44	66	6363
	3	3	3	102	140	1.51	53	6571

From this table 3 and table 4, the optimal inventory policy can be easily found by comparing $JETC(Q_i^*, k_i^*, L_i, n)$, for $i = 0, 1, 2, 3$.

Example 3. The data for example 3 are the same as example 1. Suppose that the lead time follows a free distribution and we solve for the case when α=1.25. Applying the algorithm in linear case (Free distribution), the results of the solution are illustrated in Table 4 and the corresponding curve of the minimum joint expected total cost is plotted in Figure 3.

Example 4. The data for example 2 are the same as example 4. Suppose that the lead time follows a free distribution and we solve for the case when τ=-0.5. Applying

Table 5: The optimal solutions for linear relationship case using free distribution (L_i in weeks)

α	i	L_i	n	A_i^{**}	Q_i^{**}	k_i^{**}	R_i^{**}	$JETC$
	0	8	3	200	219	1.0025	112	7844
1.25	1	6	3	160	211	1.0335	87	7531
	2	4	3	120	155	1.3031	64	7166
	3	3	3	100	154	1.3091	50	7050

the algorithm in logarithmic case (Free distribution), the results of the solution are illustrated in Table 6 and the corresponding curve of the minimum joint expected total cost is plotted in figure 4.

Table 6: The optimal solutions for linear relationship case using free distribution (L_i in weeks)

τ	i	L_i	n	A_i^{**}	Q_i^{**}	k_i^{**}	R_i^{**}	$JETC$
	0	8	3	200	219	1.0025	112	7844
-0.5	1	6	3	171	164	1.2519	91	7595
	2	4	3	131	157	1.2914	64	7212
	3	3	3	102	154	1.3091	50	7058

From table 5 and table 6, the optimal inventory policy can be easily found by comparing $JETC(Q_i^{**}, k_i^{**}, L_i, n)$, for i=0,1,2,3.

6. SENSITIVITY ANALYSIS

We consider the case that the relationship between lead time and ordering cost is linear and logarithmic. To further illustrate the model and algorithm, we now study the effects of parameter α on the optimal lot size Q^*, safety factor k^*, reorder point r^* and the minimum joint expected total cost $JETC(Q^*, k^*, L, n)$. of example 1 and example 2 using normal distribution and the effects of parameter τ on the optimal lot size Q^{**}, safety factor k^{**}, reorder point r^{**} and the minimum joint expected total cost $JETC(Q^{**}, k^{**}, L, n)$ of example 3 and example 4 using free distribution. The sensitivity analysis is performed by changing the parameters of α = 5, 2.50, 1 ,0.75 of example 1 and example 2, and τ = 0, -0.2, -0.8, -1 of example 3 and example 4.

The results are presented in Table 7 to table 10 and the corresponding curves of the minimum joint expected total cost are plotted in figures (5) to (8).

Table 7: Effects of α on optimal solution in linear function case (normal distribution)

α	i	L_i	n	A_i^*	Q_i^*	k_i^*	R_i^*	$JETC$
	0	8	3	200	143	1.30	118	6781
5.00	1	6	3	190	142	1.31	92	6661
	2	4	3	180	142	1.31	65	6578
	3	3	3	175	144	1.30	50	6632
	0	8	3	200	143	1.30	118	6781
2.50	1	6	3	180	141	1.31	92	6622
	2	4	3	160	140	1.32	65	6491
	3	3	3	150	142	1.31	50	6532
	0	8	3	200	143	1.30	118	6781
1.00	1	6	3	150	142	1.32	92	6495
	2	4	3	100	142	1.34	65	6231
	3	3	3	75	144	1.34	51	6208
	0	8	3	200	143	1.30	118	6781
0.75	1	6	3	133	136	1.33	92	6420
	2	4	3	67	130	1.36	65	6677
	3	3	3	33	130	1.36	51	6016

7. MANAGERIAL IMPLICATIONS

1. From table 7, it is interesting to observe that decreasing the value of α will result in a decrease in the Joint Expected Total Cost ($JETC$), the order quantity Q and increase in the safety factor k^*.

2. From table 8, it is interesting to observe that decreasing the value of τ will result in a decrease in the Joint Expected Total Cost ($JETC$), the order quantity Q and increase in the safety factor k^*.

3. From table 9, it is interesting to observe that decreasing the value of α will result in a decrease in the Joint Expected Total Cost ($JETC$), the order quantity Q and increase in the safety factor k^{**}.

Table 8: Effects of τ on optimal solution in logarithmic function case (normal distribution)

α	i	L_i	n	A_i^*	Q_i^*	k_i^*	R_i^*	$JETC$
	0	8	3	200	143	1.30	118	6781
0.0	1	6	3	200	143	1.30	92	6709
	2	4	3	200	144	1.30	64	6664
	3	3	3	200	147	1.29	50	6740
	0	8	3	200	143	1.30	118	6781
-0.2	1	6	3	188	142	1.31	92	6653
	2	4	3	172	141	1.31	64	6547
	3	3	3	161	143	1.30	50	6582
	0	8	3	200	143	1.30	118	6781
-0.8	1	6	3	154	138	1.32	92	6512
	2	4	3	89	133	1.34	65	6184
	3	3	3	43	131	1.35	51	6064
	0	8	3	200	143	1.30	118	6781
-1.0	1	6	3	142	137	1.33	92	6455
	2	4	3	61	129	1.36	65	6054
	3	3	3	4	127	1.37	51	5881

4. From table 10, it is interesting to observe that decreasing the value of τ will result in a decrease in the Joint Expected Total Cost ($JETC$), the order quantity Q and increase in the safety factor k^{**}.

8. CONCLUSION

Manufacturing is the backbone of any industrialized nation, particularly developing countries. Recent worldwide advances in manufacturing technologies have brought about a change in industry. Fast-changing technologies on the product front have created a need for an equally fast response from manufacturing industries. To meet these challenges, manufacturing industries have to select appropriate manufacturing strategies, product designs, manufacturing processes, work piece and tool materials, inventory management, and machinery and equipment. The selection decisions are complex, as decision making is more challenging nowadays. Decision makers in the manufacturing sector regularly face the problem of managing/producing inventory

Table 9: Effects of α on optimal solution in linear function case (free distribution)

α	i	L_i	n	A_i^{**}	Q_i^{**}	k_i^{**}	R_i^{**}	$JETC$
	0	8	3	200	219	1.0025	112	7844
5.00	1	6	3	190	214	1.0217	87	7620
	2	4	3	180	209	1.0415	61	7387
	3	3	3	175	208	1.0455	47	7325
	0	8	3	200	219	1.0025	112	7844
2.50	1	6	3	180	164	1.2519	91	7628
	2	4	3	160	159	1.2799	64	7325
	3	3	3	150	158	1.2856	50	7248
	0	8	3	200	219	1.0025	112	7844
1.00	1	6	3	150	162	1.2630	91	7514
	2	4	3	100	153	1.3151	65	7085
	3	3	3	75	151	1.3272	51	6948
	0	8	3	200	219	1.0025	112	7844
0.75	1	6	3	133	160	1.2742	91	7448
	2	4	3	67	150	1.3333	65	6948
	3	3	3	33	147	1.3521	51	6774

products in an uncertain environment. This study addressed a two-echelon supply chain problem consisting of a single vendor and a single buyer and also considered order quantity, reorder point and lead time as a decision variables.

The demand during lead time follows a normal distribution in the first model, and in the second model, the distribution free approach is applied for the lead time demand. And we considered the case where the lead time reductions and ordering cost with linear function, then considered the logarithmic functional relationship. we seek to minimize the joint expected total cost for the buyer and the vendor for both the normal distribution and the distribution free cases. A solution procedure is developed to find the optimal solution and numerical solution is presented to illustrate the proposed models. Developing the model to the single-buyer multiple-vendor, multiple-buyer single-vendor and multiple-buyer multiple-vendor systems. Another possible extension of this work can be done by assuming a discrete investment to reduce the vendor's setup cost instead of continuous investment.

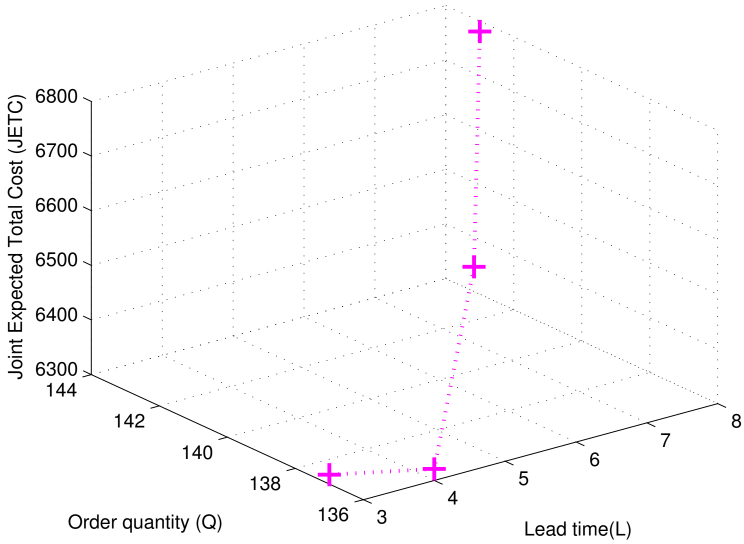


Figure 1: Graphical representation of Example 1

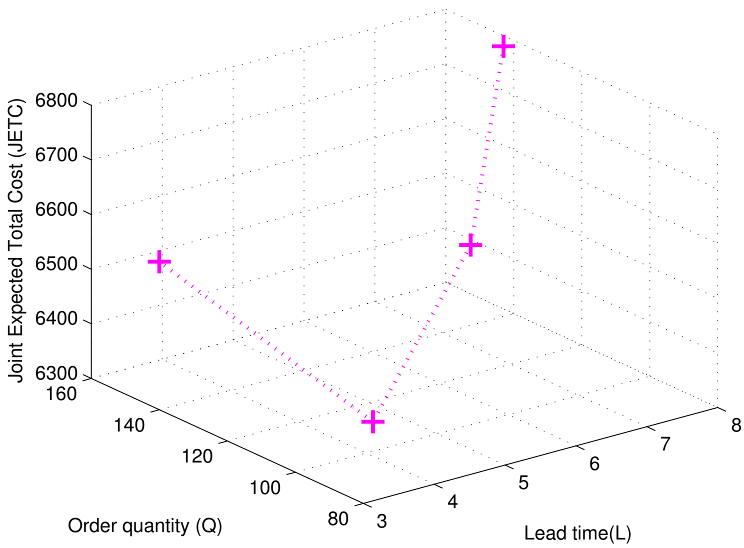


Figure 2: Graphical representation of Example 2

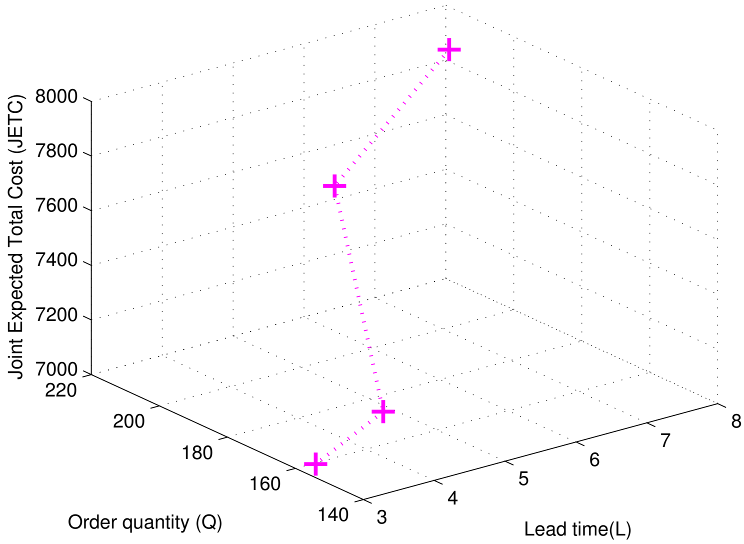


Figure 3: Graphical representation of Example 3

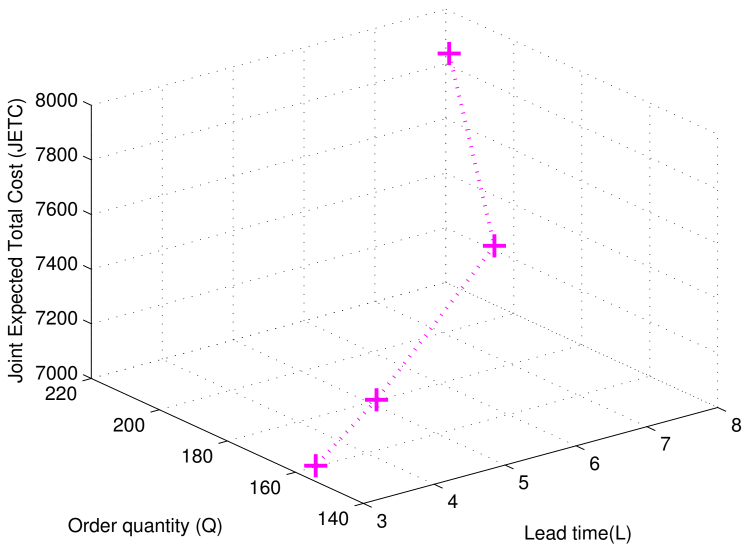


Figure 4: Graphical representation of Example 4

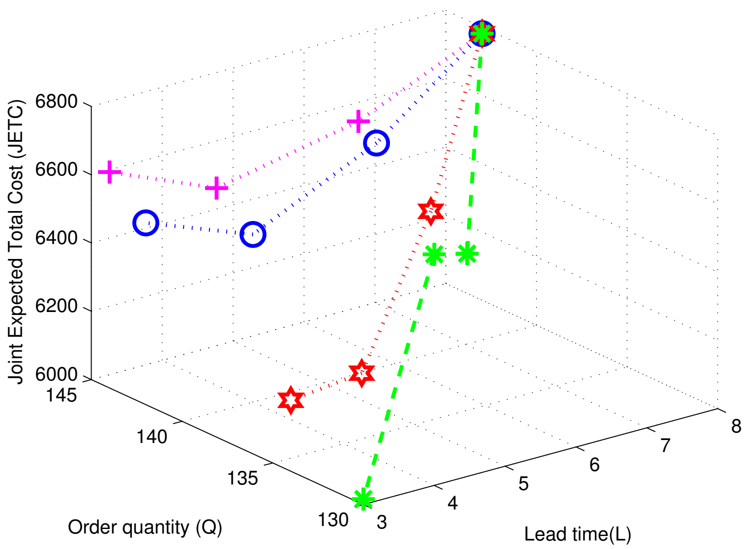


Figure 5: Graphical representation of Table 7

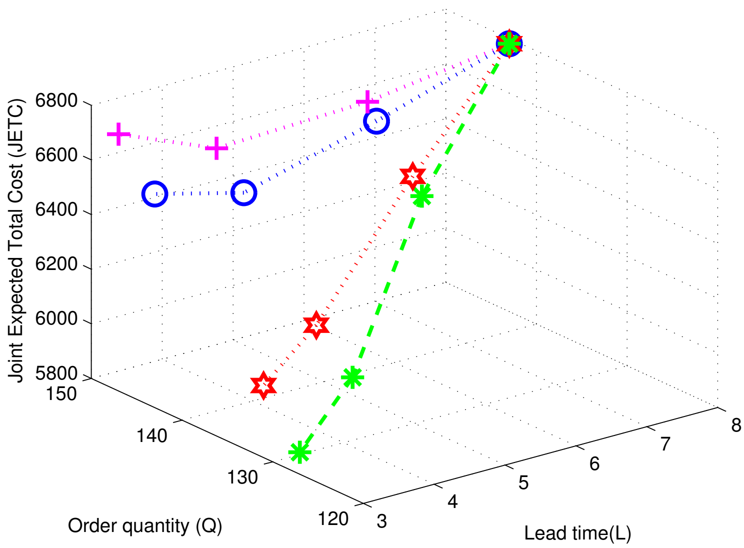


Figure 6: Graphical representation of Table 8

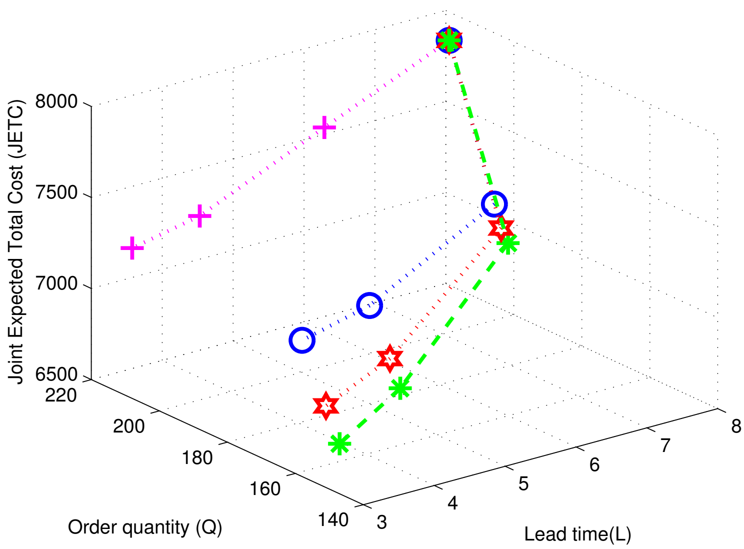


Figure 7: Graphical representation of Table 9

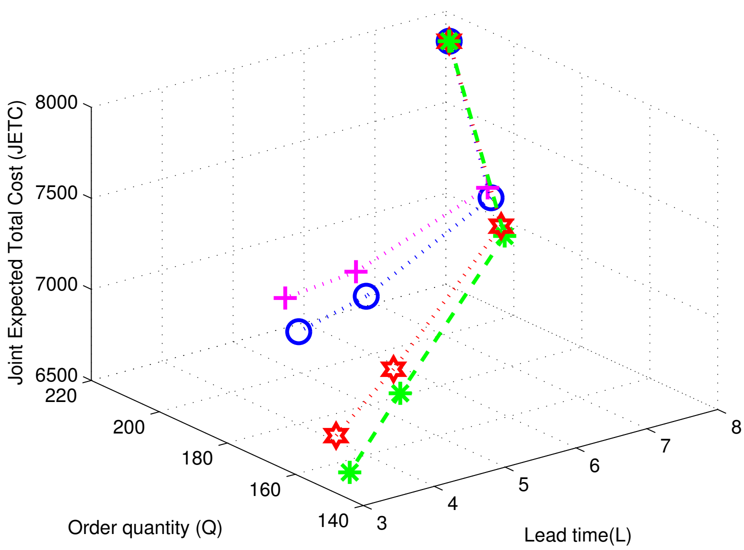


Figure 8: Graphical representation of Table 10

Table 10: Effects of τ on optimal solution in logarithmic function case (free distribution)

α	i	L_i	n	A_i^{**}	Q_i^{**}	k_i^{**}	R_i^{**}	$JETC$
	0	8	3	200	219	1.0025	112	7844
0.0	1	6	3	200	166	1.2410	91	7705
	2	4	3	200	163	1.2574	64	7481
	3	3	3	200	163	1.2574	50	7442
	0	8	3	200	219	1.0025	112	7844
-0.2	1	6	3	118	165	1.2464	91	7659
	2	4	3	172	160	1.2742	64	7372
	3	3	3	161	159	1.2799	50	7291
	0	8	3	200	219	1.0025	118	7844
-0.8	1	6	3	154	162	1.2630	91	7529
	2	4	3	89	152	1.3211	65	7040
	3	3	3	43	148	1.3458	51	6815
	0	8	3	200	219	1.0025	112	7844
-1.0	1	6	3	142	161	1.2686	91	7482
	2	4	3	61	150	1.3333	65	6925
	3	3	3	4	144	1.3714	51	6648

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APPENDIX

For a given value of L, we first obtain the Hessian Matrix $\mathbf{H}$ as follows:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 JETC(Q,k,L,n)}{\partial Q^2} & \frac{\partial^2 JETC(Q,k,L,n)}{\partial Q \partial k} \\ \frac{\partial^2 JETC(Q,k,L,n)}{\partial k \partial Q} & \frac{\partial^2 JETC(Q,k,L,n)}{\partial Q^2} \end{bmatrix}$$

$$\begin{aligned} \frac{\partial^2 EAC(Q,k,L,n)}{\partial Q^2} &= \frac{2D}{Q^3}[A(L) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)] \\ \frac{\partial^2 JETC(Q,k,L,n)}{\partial Q \partial k} &= \frac{D}{Q^2}\pi\sigma\sqrt{L}[\Phi(k) - 1] \\ \frac{\partial^2 JETC(Q,k,L,n)}{\partial k \partial Q} &= \frac{D}{Q^2}\pi\sigma\sqrt{L}[\Phi(k) - 1] \\ \frac{\partial^2 JETC(Q,k,L,n)}{\partial k^2} &= \frac{D}{Q}\pi\sigma\sqrt{L}\phi(k) \end{aligned}$$

Then we proceed by evaluating the principal minor determinant of $\mathbf{H}$.
The first principal minor determinant of $\mathbf{H}$ is

$$|H_{11}| = \frac{\partial^2 EAC(Q,k,L,n)}{\partial Q^2} = \frac{2D}{Q^3}[A(L) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)] > 0$$

The second principle minor determinant of $\mathbf{H}$ is:

$$\begin{aligned} |H_{22}| &= \begin{bmatrix} \frac{\partial^2 JETC(Q,k,L,n)}{\partial Q^2} & \frac{\partial^2 JETC(Q,k,L,n)}{\partial Q \partial k} \\ \frac{\partial^2 JETC(Q,k,L,n)}{\partial k \partial Q} & \frac{\partial^2 JETC(Q,k,L,n)}{\partial k^2} \end{bmatrix} \\ &= \frac{2D}{Q^4}\pi\sigma\sqrt{(}k)\psi(k)[A(L) + \frac{S}{n} + \pi\sigma\sqrt{L}\psi(k) + C(L)] - \frac{D^2}{Q^4}\pi^2\sigma^2 L[\psi(k) - 1]^2 \\ &= \frac{2D}{Q^4}\pi\sigma\sqrt{(}k)\psi(k)[A(L) + \frac{S}{n} + C(L)] + \frac{D^2}{Q^4}\pi^2\sigma^2 L[2\psi(k)\phi(k) - (\Phi(k) - 1)^2] \\ &> 0. \end{aligned}$$

as $\phi(k)$, $\psi(k) > 0$ and $2\phi(k)\psi(k) - (\Phi(k) - 1)^2 > 0$ for all $k > 0$. [17].

We see that all the principal minors of the Hessian Matrix are positive. Hence, the Hessian Matrix $\mathbf{H}$ is positive definite at (Q, k).

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