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MATHEMATICAL DESIGN OF PROCESS OF WATER TREATMENT BY FILTER-CLARIFIER WITH LAYER OF HANGING UP SEDIMENT

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ABSTRACT: The mathematical model of water treatment is formed and analyzed in filter-clarifier, taking into account the influence of dose of reagent and irreversible coagulation of admixture particles. The algorithm of the numerical-asymptotic approaching of decision of corresponding modeling nonlinear spatial problem is built for the system of differential equations as "convection-diffusion-mass-transfer". The computer experiment is conducted on this basis.

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1. INTRODUCTION

At preparation of drinking-water from the natural sources of the centralized water-supply the system that consists of clarifiers and filter devices is used. These devices are compact incorporated in setting with the floating filter loading, that showed positive results. The maximal degree of natural water treatment depends, as known, from the dose of reagent.

In-process [1] for setting that combines the processes of clearing up of water and filtration, executed after a patent [2], models of natural water treatment are got, both in clarifier and in a filter. For filter, influence of dose of reagent is taken into account, and there is not such account for clarifier. In [1] a quantitative analysis of influence

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of dose of reagent (to the coagulant) is to work of clarifier is absent. An author considers that the gotten criterion of separation is just "at the normal terms of work of clarifier: at a correct trouble-free dosage to the coagulant, in default of laying out of flakes of coagulated suspension and frequent, sharp vibrations of the efficiency and temperatures of water.

At water treatment by the colloid-dispersible system, the process of division of liquid and hard phases intensifies due to enlargement of admixture particles in aggregates by means of reagent and unreagent methods. The first methods include the methods of introduction to the water that clear up, flocculants, oxidants, regulators of pH, mineral pollutants. The second methods include stirring of water finished by coagulants; various methods of introduction of coagulants to the water; combination of coagulation by coagulants that are hydrolyzing, with the physical methods of coagulation are treatments of water in the magnetic and electric fields, by ionizing irradiation, ultrasound, and others like that. Huge experimental material is accumulated on these questions. However his authenticity causes doubts often. I n particular, E.D. Babencov paid attention on the necessity of very careful approach for an experimental fund: "Attracting in the selective order experimental material that touches these separate questions of coagulation it is possible t o ground or refute anything actually even theoretical presentations that conflict with each other" [3], p.112. The aforesaid testifies to the necessity of development of theory of processes of water treatment from only positions to the higher mathematical level, than it is accepted in a dispersoidology, so as, obviously, in general case difficult processes can not be adequately enough described by elementary methods [4-10].

Coming from higher said, the aim of this work is forming and analysis of mathematical model of natural water treatment in filter-clarifier taking into account influence of dose of reagent and irreversible coagulation of admixture particles.

2. STATEMENT OF THE PROBLEM

We will consider the curvilinear hexahedral filter of Gz = ABCDA*B*C*D*, limited to smooth ,orthogonal inter se in angular points and ribs, by equipotential surfaces

$$ABCD = \{(x,y,z) : f_1(x,y,z) = 0\} = \{(x,y,z) : z = f_1^*(x,y)\},$$

$$A_*B_*C_*D_* = \{(x,y,z) : f_2(x,y,z) = 0\} = \{(x,y,z) : z = f_2^*(x,y)\},$$

where $f_1(x,y,f_1^*(x,y)) = 0)$, $BCC_*B_* = \{z : f_4(x,y,z) = 0\}$ and surfaces of flow $ADD_*A_* = \{z : f_3(x,y,z) = 0\}$, $ABB_*A_* = \{z : f_5(x,y,z) = 0\}$, $CDD_*C_* =$

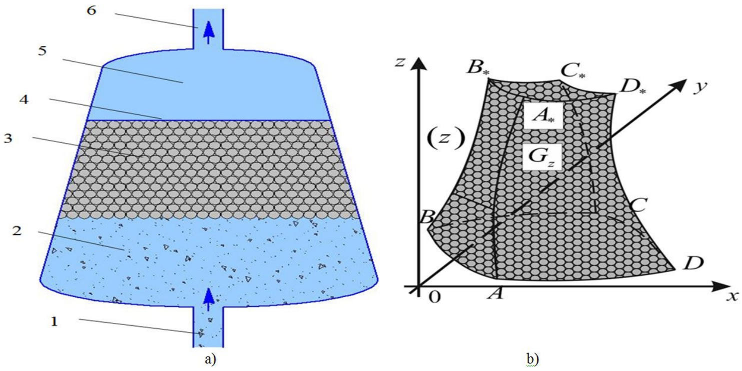

Figure 1: Schematic image of filter-clarifier : a) transversal cut of filter; b) corresponding spatial physical area of Gz (1 is a serve of initial water, 2 – distributive system, 3 – the foam polystyrene filing up, 4 is a holding grate, 5 is upper filter space, 6 is taking of clean water)

$\{z : f_6(x,y,z) = 0\}$ (see Fifure 1 (b)), as geometrical interpretation of filter-clarifier with the layer of hanging up sediment (see Figure 1 (a)), see [5].

Assume (see [4], [5]), that the particles of contamination of admixtures of substance can go across from one state in other (processes of fascination-tearing away, persorption-desorption), here the concentrations of contamination influence on descriptions of corresponding environment. Corresponding process of filtration, as generalization [6], will describe such model problem:

$$\begin{cases} \dfrac{\partial\left(\left(1-\varepsilon\left(P+\bar{a}C\right)\right)C\right)}{\partial t} = -\gamma P\left(M,t-\tau\right)C\left(M,t-\tau\right) \\ \qquad -\vec{v}\cdot\vec{\nabla}C + D_C\dfrac{\partial^2 C}{\partial x^2}, \\ \dfrac{\partial P}{\partial t} = \alpha U\left(M,t-\tau\right) - \vec{v}\cdot\vec{\nabla}P \\ \qquad - g\rho\left(1-\varepsilon\left(P+\bar{a}C\right)\right)\left(z-f_1^*\left(x,y\right)\right), \\ \dfrac{\partial U}{\partial t} = -\beta P\left(M,t-\tau\right) - \vec{v}\cdot\vec{\nabla}U \\ \qquad - D_U\dfrac{\partial^2 U}{\partial x^2},\ \left(x,y,z,t\right)\in G = G_z\times\left(0,\infty\right). \end{cases} \tag{1}$$

$$C\left|_{ABCD}\right. = C_*\left(M,t\right), \qquad P\left|_{ABCD}\right. = P_*\left(M,t\right),$$

$$U\left|_{ABCD}\right. = U_*\left(M,t\right), \qquad \frac{\partial C}{\partial\vec{n}}\left|_{CDD_*C_*}\right. = 0,$$

$$\frac{\partial C}{\partial\vec{n}}\left|_{ADD_*A_*\cup BCC_*B_*\cup ABB_*A_*\cup CDC_*D_*}\right. = 0, \qquad \frac{\partial P}{\partial\vec{n}}\left|_{CDD_*C_*}\right. = 0,$$

$$\frac{\partial P}{\partial\vec{n}}\left|_{ADD_*A_*\cup BCC_*B_*\cup ABB_*A_*\cup CDC_*D_*}\right. = 0, \qquad \frac{\partial U}{\partial\vec{n}}\left|_{CDD_*C_*}\right. = 0,$$

$$\frac{\partial U}{\partial\vec{n}}\left|_{ADD_*A_*\cup BCC_*B_*\cup ABB_*A_*\cup CDC_*D_*}\right. = 0, \qquad C\left(x,y,z,\tilde{t}\right) = C_0^0\left(x,y,z,\tilde{t}\right),$$

$$P\left(x,y,z,\tilde{t}\right) = P_0^0\left(x,y,z,\tilde{t}\right),$$

$$U\left(x,y,z,\tilde{t}\right) = U_0^0\left(x,y,z,\tilde{t}\right), \quad x \le L, \quad -\tau \le \tilde{t} \le 0; \tag{2}$$

$$\vec{v} = \kappa\left(P\right)\nabla\varphi, \nabla\cdot\,\vec{v} = 0, \tag{3}$$

$$\varphi\left|_{ABCD}\right. = \varphi_*, \varphi\left|_{A_*B_*C_*D_*}\right. = \varphi^*, \tag{4}$$

where $C\left(M,t\right)$, $P\left(M,t\right)$, $U\left(M,t\right)$ accordingly the concentrations of admixtures, flake sand substances for creation of flakes in a filtration flow [6] in the internal point (x, y, z) of swap area in the moment of time of t; $\vec{\nabla}$ is an operator of Hamilton, γ is a coefficient that characterizes fascination of muddy particles by flakes, α, β accordingly coefficients that characterize the amount of reagent for a flocculation and flakes form a reagent for time unit, g is an acceleration of the gravity, ρ is density, $\bar{a}$ is a coefficient of transformation (contaminations in flakes), D_C, D_U the coefficients of diffusion, where,

$$D_C = b_C\varepsilon, \quad D_U = b_U\varepsilon, \quad 0 < b_C \le 1,\ 0 < b_U \le 1,$$

ε are small parameters, $\tau > 0$ a delay time. Will mark, that functions $C_*\left(M,t\right)$, $P_*\left(M,t\right)$, $U_*\left(M,t\right)$, $C_0^0(M,t)$, $P_0^0(M,t)$, $U_0^0(M,t)$ are smooth enough and concerted in the angular points of area of their set up. Additionally we consider that functions $C_0^0(M,t)$, $P_0^0(M,t)$, $U_0^0(M,t)$ at $t = -\tau$ and $t = 0$ satisfy terms that provide necessary for realization of further expositions smoothness of decision

$$C = C(M,t), \quad P = P(M,t), \quad U = U(M,t)$$

of this problem at $t = \tau n$ $(n = 1,2,...)$; M is a hurrying point of corresponding surface; φ is filtration potential $(0 < \varphi_* \le \varphi \le \varphi^* < \infty\ )$; $\vec{v}\left(v_x, v_y, v_z\right)$ is a vector of speed of filtration ($|\vec{v}| > v_* >> \varepsilon$); $\vec{n}$ an external normal to the corresponding surface.

We will accept, that problem (3), (4) on a spatial conformal reflection

$$G_w \mapsto G_z$$

($G_w = \{w = (\varphi,\psi,\eta):\ \varphi_* < \varphi < \varphi^*,\ 0 < \psi < Q_*, 0 < \eta < Q^*\}$ corresponding Gz area of complex potential) at some middle meaning κ is solved, in particular, dynamic net and field of speed $\vec{v}$ are built, filtration expense is calculated of Q=Q_*Q^*, see [4, 5]. Then, after overdone replacement of variables

$$x = x\left(\varphi,\psi,\eta\right), \quad y = y\left(\varphi,\psi,\eta\right), \quad z = z\left(\varphi,\psi,\eta\right)$$

in the system (1) and terms (2), come to the corresponding problem for the area of $G_W \times (0\ \infty)$ (see, for example [5]):

$$
\left\{
\begin{aligned}
\frac{\partial\left(\left(1-\varepsilon\left(p+\bar{a}c\right)\right)c\right)}{\partial t} =& -\gamma pc - v^2\frac{\partial c}{\partial\varphi} \\
&+\varepsilon d_c\left(v^2\frac{\partial^2 c}{\partial\varphi^2}+b_{c1}\frac{\partial^2 c}{\partial\psi^2}+b_{c2}\frac{\partial^2 c}{\partial\eta^2}+d_{c1}\frac{\partial c}{\partial\psi}+d_{c2}\frac{\partial c}{\partial\eta}\right), \\
\frac{\partial p}{\partial t} =& \alpha u - v^2\frac{\partial p}{\partial\varphi} - g\rho\left(1-\varepsilon\left(p+\bar{a}c\right)\right)\left(\varphi-\varphi_*\right), \\
\frac{\partial u}{\partial t} =& -\beta p - v^2\frac{\partial u}{\partial\varphi} \\
&+\varepsilon d_u\left(v^2\frac{\partial^2 u}{\partial\varphi^2}+b_{u1}\frac{\partial^2 u}{\partial\psi^2}+b_{u2}\frac{\partial^2 u}{\partial\eta^2}+d_{u1}\frac{\partial u}{\partial\psi}+d_{u2}\frac{\partial u}{\partial\eta}\right),
\end{aligned}
\right. \tag{5}
$$

$$
\begin{aligned}
c(\varphi_*,\psi,\eta,t) =& c_*\left(\psi,\eta,t\right), \\
c_\varphi(\varphi^*,\psi,\eta,t) =& 0, \\
c_\psi(\varphi,0,\eta,t) =& c_\psi(\varphi,Q_*,\eta,t) = c_\eta(\varphi,\psi,0,t) = c_\eta(\varphi,\psi,Q^*,t) = 0, \\
p(\varphi_*,\psi,\eta,t) =& p_*\left(\psi,\eta,t\right), \\
p_\varphi(\varphi^*,\psi,\eta,t) =& 0, \\
p_\psi(\varphi,0,\eta,t) =& p_\psi(\varphi,Q_*,\eta,t) = p_\eta(\varphi,\psi,0,t) = p_\eta(\varphi,\psi,Q^*,t) = 0, \\
u(\varphi_*,\psi,\eta,t) =& u_*\left(\psi,\eta,t\right), \\
u_\varphi(\varphi^*,\psi,\eta,t) =& 0, \\
u_\psi(\varphi,0,\eta,t) =& u_\psi(\varphi,Q_*,\eta,t) = u_\eta(\varphi,\psi,0,t) = u_\eta(\varphi,\psi,Q^*,t) = 0, \\
c(\varphi,\psi,\eta,0) =& c_0^0\left(\varphi,\psi,\eta\right), \rho(\varphi,\psi,\eta,0) = \rho_0^0\left(\varphi,\psi,\eta\right),
\end{aligned}
$$

$$
u(\varphi,\psi,\eta,0) = u_0^0\left(\varphi,\psi,\eta\right), \tag{6}
$$

where

$$
\begin{aligned}
c =& c\left(\varphi,\psi,\eta,t\right) = C\left(x\left(\varphi,\psi,\eta\right), y\left(\varphi,\psi,\eta\right), z\left(\varphi,\psi,\eta\right), t\right), \\
\rho =& \rho\left(\varphi,\psi,\eta,t\right) = P\left(x\left(\varphi,\psi,\eta\right), y\left(\varphi,\psi,\eta\right), z\left(\varphi,\psi,\eta\right), t\right), \\
u =& u\left(\varphi,\psi,\eta,t\right) = U\left(x\left(\varphi,\psi,\eta\right), y\left(\varphi,\psi,\eta\right), z\left(\varphi,\psi,\eta\right), t\right).
\end{aligned}
$$

3. ASYMPTOTIC OF THE DECISION

The problem (5)-(6) (with the delay τ) we will bring together to the sequence of tasks without a delay (on temporal intervals $[(n-1)\tau, n\tau]$, $n = 1, 2, ...$), see [5]:

$$
\begin{cases}
\left(1-\varepsilon\left(p_t^{[n]}+\bar{\alpha}c_t^{[n]}\right)c_t^{[n]}\right)=-\gamma p_{n\tau}c_{n\tau}-v^2c_{\varphi}^{[n]}\\
+\varepsilon d_c\left(v^2\dfrac{\partial^2 c}{\partial\varphi^2}+b_{c1}\dfrac{\partial^2 c}{\partial\psi^2}+b_{c2}\dfrac{\partial^2 c}{\partial\eta^2}+d_{c1}\dfrac{\partial c}{\partial\psi}+d_{c2}\dfrac{\partial c}{\partial\eta}\right),\\
p_t^{[n]}=\alpha u_{n\tau}-v^2p_{\varphi}^{[n]}-g\rho\left(1-\varepsilon\left(p^{[n]}+\bar{\alpha}c^{[n]}\right)c^{[n]}\right)(\varphi-\varphi_*),\\
u_t^{[n]}=-\beta p_{n\tau}-v^2u_{\varphi}^{[n]}\\
+\varepsilon d_u\left(v^2\frac{\partial^2 u}{\partial\varphi^2}+b_{u1}\frac{\partial^2 u}{\partial\psi^2}+b_{u2}\frac{\partial^2 u}{\partial\eta^2}+d_{u1}\frac{\partial u}{\partial\psi}+d_{u2}\frac{\partial u}{\partial\eta}\right),\\
c^{[n]}(\bar{\varphi}_*,\psi,\eta,t)=c_*(\psi,\eta,t),\quad c_{n\tau}(\bar{\varphi}_*,\psi,\eta,t)=c^{[n]}(\bar{\varphi}_*,\psi,\eta,t-\tau)\\
=c^{[n-1]}(\bar{\varphi}_*,\psi,\eta,t-\tau),\ c^{[0]}(\bar{\varphi}_*,\psi,\eta,0)=c_0^0(\bar{\varphi}_*,\psi,\eta,0),\\
p^{[n]}(\bar{\varphi}_*,\psi,\eta,t)=p_*(\psi,\eta,t),\quad p_{n\tau}(\bar{\varphi}_*,\psi,\eta,t)=p^{[n]}(\bar{\varphi}_*,\psi,\eta,t-\tau)\\
p^{[n-1]}(\bar{\varphi}_*,\psi,\eta,t-\tau),\ p^{[0]}(\bar{\varphi}_*,\psi,\eta,0)=p_0^0(\bar{\varphi}_*,\psi,\eta,0),\\
u^{[n]}(\bar{\varphi}_*,\psi,\eta,t)=u_*(\psi,\eta,t),\quad u_{n\tau}(\bar{\varphi}_*,\psi,\eta,t)=u^{[n]}(\bar{\varphi}_*,\psi,\eta,t-\tau)\\
=u^{[n-1]}(\bar{\varphi}_*,\psi,\eta,t-\tau),\ u^{[0]}(\bar{\varphi}_*,\psi,\eta,0)=u_0^0(\bar{\varphi}_*,\psi,\eta,0).
\end{cases}
\tag{7}
$$

The decision of task (7) with exactness $(O(\varepsilon^n))$ we search as asymptotic rows [4], [5]:

$$
c^{[n]}=c_0^{[n]}+\sum_{j=1}^{n}\varepsilon^j c_j^{[n]}+\sum_{j=0}^{n}\varepsilon^j\Pi_j^{[n]}+\sum_{j=0}^{n}\varepsilon^j\breve{\Pi}_j^{[n]}
+\sum_{j=0}^{n+1}\varepsilon^{j/2}\tilde{\Pi}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\overline{\overline{\Pi}}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\widehat{\Pi}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\underline{\underline{\Pi}}_j^{[n]}+R_c^{[n]},
$$

$$
p^{[n]}=p_0^{[n]}+\sum_{j=1}^{n}\varepsilon^j p_j^{[n]}+\sum_{j=0}^{n}\varepsilon^j\bar{P}_j^{[n]}+\sum_{j=0}^{n}\varepsilon^i\breve{P}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\tilde{P}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\overline{\overline{P}}_j^{[n]}
+\sum_{j=0}^{n+1}\varepsilon^{j/2}\widehat{P}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\underline{\underline{P}}_j^{[n]}+R_p^{[n]}, \tag{8}
$$

$$
u^{[n]}=u_0^{[n]}+\sum_{j=1}^{n}\varepsilon^j u_j^{[n]}+\sum_{j=0}^{n}\varepsilon^j\bar{U}_j^{[n]}+\sum_{j=0}^{n}\varepsilon^i\breve{U}_j^{[n]}
+\sum_{j=0}^{n+1}\varepsilon^{j/2}\tilde{U}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\overline{\overline{U}}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\widehat{U}_j^{[n]}+\sum_{j=0}^{n+1}\varepsilon^{j/2}\underline{\underline{U}}_j^{[n]}+R_u^{[n]},
$$

where $R_c^{[n]}(\varphi,\psi,\eta,t,\varepsilon)$, $R_p^{[n]}(\varphi,\psi,\eta,t,\varepsilon)$, $R_u^{[n]}(\varphi,\psi,\eta,t,\varepsilon)$ are remaining members, $c_j^{[n]}(\varphi,\psi,\eta,t)$, $p_j^{[n]}(\varphi,\psi,\eta,t)$, $u_j^{[n]}(\varphi,\psi,\eta,t)$ are members of regular part of asymptotic $(j=\overline{0,\ n})$; $\Pi_j^{[n]}(\xi,\psi,\eta,t)$, $\bar{P}_j^{[n]}(\xi,\psi,\eta,t)$, $\bar{U}_j^{[n]}(\xi,\psi,\eta,t)$ are functions of the type of to the border layer in the round on temporal intervals $\varphi=\varphi^*$ (corrections are on an exit from filter) $\left(j=\overline{0,2}\right)$ $\breve{\Pi}_j^{[n]}\left(\breve{\xi},\psi,\eta,t\right)$, $\breve{P}_j^{[n]}\left(\breve{\xi},\psi,\eta,t\right)$, $\breve{U}_j^{[n]}\left(\breve{\xi},\psi,\eta,t\right)$ - in the round (amendments are on the entrance in a filter) $\left(j=\overline{0,2}\right)$, and functions $\tilde{\Pi}_j^{[n]}\left(\varphi,\tilde{\psi},\eta,t\right)$, $\overline{\overline{\Pi}}_j^{[n]}(\varphi,\tilde{\tilde{\psi}},\eta,t)$, $\widehat{\Pi}_j^{[n]}(\varphi,\psi,\tilde{\eta},t)$, $\underline{\underline{\Pi}}_j^{[n]}(\varphi,\psi,\tilde{\tilde{\eta}},t)$, $\tilde{P}_j\left(\varphi,\tilde{\psi},\eta,t\right)$,

$\overline{\overline{P}}_j\left(\varphi,\tilde{\tilde{\psi}},\eta,t\right)$, $\widehat{P}_j\left(\varphi,\psi,\tilde{\eta},t\right)$, $\underline{\underline{P}}_j\left(\varphi,\psi,\tilde{\tilde{\eta}},t\right)$, $\tilde{U}_j\left(\varphi,\tilde{\psi},\eta,t\right)$, $\overline{\overline{U}}_j\left(\varphi,\tilde{\tilde{\psi}},\eta,t\right)$, $\widehat{U}_j^{[n]}(\varphi,\psi,\tilde{\eta},$ $t)$, $\underline{\underline{U}}_j^{[n]}\left(\varphi,\psi,\tilde{\tilde{\eta}},t\right)$, $\left(j=\overline{0,3}\right)$ – in rounds ψ=0, ψ=Q*, η=0, η=Q*, ψ=Q*, η=0, η=Q* (corrections are on the lateral walls of filter), accordingly; $\tilde{\psi}=(Q_*-\psi)/\varepsilon$, $\xi=(\varphi^*-\varphi)/\varepsilon$, $\breve{\xi}=(\varphi-\varphi_*)/\varepsilon$, $\overline{\overline{\psi}}=\psi/\varepsilon$, $\tilde{\eta}=(Q^*-\eta)/\varepsilon$, $\overline{\overline{\eta}}=\eta/\varepsilon$.

Similarly [5], as a result of substitution (8) in (7), application of standard "procedure of equating", and decision of corresponding intermediate tasks we have:

$$c_0^{[n]}=\begin{cases} c_*\left(\psi,\eta,t-f\right)exp\left[-\gamma\int\limits_{\varphi_*}^{\varphi}\dfrac{p_{n\tau}c_{n\tau}d\tilde{\varphi}}{v^2\left(\tilde{\varphi},\psi,\eta\right)}\right], & t\geq f,\\ c_0^0\left(f^{-1}\left(f-t,\psi,\eta\right),\psi,\eta\right)exp\left[-\gamma p_{n\tau}c_{n\tau}t\right], & t<f,\end{cases}$$

$$c_j^{[n]}=\begin{cases} e^{-\lambda_{c1}}\int\limits_{\varphi_0}^{\varphi}\dfrac{\bar{C}_j^{[n]}\left(s,\psi,\eta,f\left(s,\psi,\eta\right)-f+t\right)}{v^2\left(s,\psi,\eta\right)}e^{\lambda_{c2}}ds, & t\geq f,\\ -e^{-\lambda_{c1}}\int\limits_{(n-1)\tau}^{t}\dfrac{\bar{C}_j^{[n]}\left(f^{-1}\left(s+f-t,\psi,\eta\right),\psi,\eta,s\right)}{\bar{c}_j^{[n]}\left(f^{-1}\left(s+f-t,\psi,\eta\right),\psi,\eta\right)}e^{\lambda_{c2}}ds, & t<f,\end{cases}$$

$$p_0^{[n]}=\begin{cases} p_*\left(\psi,\eta,t-f\right)exp\left[\alpha\int\limits_{\varphi_*}^{\varphi}\dfrac{u_{n\tau}d\tilde{\varphi}}{v^2\left(\tilde{\varphi},\psi,\eta\right)}\right], & t\geq f,\\ p_0^0\left(f^{-1}\left(f-t,\psi,\eta\right),\psi,\eta\right)exp\left[\alpha u_{n\tau}t\right], & t<f,\end{cases}$$

$$p_j^{[n]}=\begin{cases} e^{-\lambda_{p1}}\int\limits_{\varphi_0}^{\varphi}\dfrac{\bar{P}_j^{[n]}\left(s,\psi,\eta,f\left(s,\psi,\eta\right)-f+t\right)}{v^2\left(s,\psi,\eta\right)}e^{\lambda_{p2}}ds, \quad t\geq f,\\ -e^{-\lambda_{p1}}\times\\ \times\int\limits_{(n-1)\tau}^{t}\bar{P}_j^{[n]}\left(f^{-1}\left(s+f-t,\psi,\eta\right),\psi,\eta,s\right)e^{\lambda_{p2}}ds,\\ t<f,\end{cases}$$

$$u_0^{[n]}=\begin{cases} u_*\left(\psi,\eta,t-f\right)exp\left[-\beta\int\limits_{\varphi_*}^{\varphi}\dfrac{p_{n\tau}d\tilde{\varphi}}{v^2\left(\tilde{\varphi},\psi,\eta\right)}\right], & t\geq f,\\ u_0^0\left(f^{-1}\left(f-t,\psi,\eta\right),\psi,\eta\right)exp\left[-\beta p_{n\tau}t\right], & t<f,\end{cases}$$

$$u_j^{[n]}=\begin{cases} e^{-\lambda_{1}}\int\limits_{\varphi_0}^{\varphi}\dfrac{\bar{U}_j^{[n]}\left(s,\psi,\eta,f\left(s,\psi,\eta\right)-f+t\right)}{v^2\left(s,\psi,\eta\right)}e^{\lambda_{2}}ds, \quad t\geq f,\\ -e^{-\lambda_{1}}\times\\ \times\int\limits_{(n-1)\tau}^{t}\bar{U}_j^{[n]}\left(f^{-1}\left(s+f-t,\psi,\eta\right),\psi,\eta,s\right)e^{\lambda_{2}}ds,\\ t<f,\end{cases}$$

where

$$
\begin{aligned}
\bar{c}_j^{[n]} &= \left(\frac{\partial p_{j-1}^{[n]}}{\partial t} + \bar{a} \frac{\partial c_{j-1}^{[n]}}{\partial t} \right) \frac{\partial c_{j-1}^{[n]}}{\partial t} \qquad \left(j = \overline{2, n} \right), \\
\bar{C}_j^{[n]} &= -\gamma p_{j-1}^{[n]} c_{j-1}^{[n]} \\
&\quad + d_{ci} \left(v^2 \frac{\partial^2 c_j^{[n]}}{\partial \varphi^2} + b_{c1} \frac{\partial^2 c_j^{[n]}}{\partial \psi^2} + b_{c2} \frac{\partial^2 c_j^{[n]}}{\partial \eta^2} + d_{c1} \frac{\partial c_j^{[n]}}{\partial \psi} + d_{c2} \frac{\partial c_j^{[n]}}{\partial \eta} \right), \\
\lambda_{c1} &= -\gamma \int_{\varphi_0}^{\varphi} \frac{p_{j-1}^{[n]} \left(s, \psi, \eta, \tilde{f} \right) c_{j-1}^{[n]} \left(s, \psi, \eta, \tilde{f} \right)}{v^2 (s, \psi, \eta)} ds, \\
\lambda_{c2} &= -\gamma \int_{(n-1)\tau}^{t} \frac{p_{j-1}^{[n]} \left(\bar{f} \right) c_{j-1}^{[n]} \left(\bar{f} \right)}{\bar{c}_j^{[n]} \left(f^{-1} \left(\tilde{t} + f - t, \psi, \eta \right), \psi, \eta \right)} d\tilde{s}, \\
\bar{P}_j^{[n]} (\varphi, \psi, \eta, t) &= \alpha u_{j-1}^{[n]} - g\rho \left(p_{j-1}^{[n]} + \bar{a} c_{j-1}^{[n]} \right) c_{j-1}^{[n]} (\varphi - \varphi_*), \\
\lambda_{p1} (\varphi, \psi, \eta, t) &= \alpha \int_{\varphi_0}^{\varphi} \frac{u_{j-1} \left(s, \psi, \eta, \tilde{f} \right)}{v^2 (s, \psi, \eta)} ds, \\
\lambda_{p2} (\varphi, \psi, \eta, t) &= \alpha \int_{(n-1)\tau}^{t} u_{j-1}^{[n]} \left(\bar{f} \right) d\tilde{s}, \\
\bar{C}_j^{[n]} &= -\beta p_{j-1}^{[n]} \\
&\quad + d_{ui} \left(v^2 \frac{\partial^2 u_j^{[n]}}{\partial \varphi^2} + b_{u1} \frac{\partial^2 u_j^{[n]}}{\partial \psi^2} + b_{u2} \frac{\partial^2 u_j^{[n]}}{\partial \eta^2} + d_{u1} \frac{\partial u_j^{[n]}}{\partial \psi} + d_{u2} \frac{\partial u_j^{[n]}}{\partial \eta} \right), \\
\lambda_{u1} (\varphi, \psi, \eta, t) &= -\beta \int_{\varphi_0}^{\varphi} \frac{p_{j-1}^{[n]} \left(s, \psi, \eta, \tilde{f} \right)}{v^2 (s, \psi, \eta)} ds, \\
\lambda_{u2} (\varphi, \psi, \eta, t) &= -\beta \int_{(n-1)\tau}^{t} p_{j-1}^{[n]} \left(\bar{f} \right) d\tilde{s}, \\
\tilde{f} &= f (\tilde{\varphi}, \psi, \eta) + t - f, \\
\bar{f} &= f^{-1} (\tilde{s} + f (\varphi, \psi, \eta) - t, \psi, \eta), \psi, \eta, \tilde{s}, \\
f \left(\varphi, \bar{\psi}, \bar{\eta} \right) &= \int_{\varphi_0}^{\varphi} \frac{ds}{v^2 \left(s, \bar{\psi}, \bar{\eta} \right)},
\end{aligned}
$$

is time-of-passing by the corresponding particle of way from point

$$\left(x \left(\varphi_*, \bar{\psi}, \bar{\eta} \right), y \left(\varphi_*, \bar{\psi}, \bar{\eta} \right), z \left(\varphi_*, \bar{\psi}, \bar{\eta} \right) \right) \in ABB_* A_*$$

to-point $\left(x \left(\varphi, \bar{\psi}, \bar{\eta} \right), y \left(\varphi, \bar{\psi}, \bar{\eta} \right), \; z \left(\varphi, \bar{\psi}, \bar{\eta} \right) \right) \in G_{\mathrm{z}}$ along the corresponding line of flow (as crossing some two surfaces, $\psi (x, y, z) = \bar{\psi}$, $0 \leq \bar{\psi} \leq Q_*$, $\eta (x, y, z) = \bar{\eta}$,

$0 \leq \bar{\eta} \leq Q^{*}$), f^{-1} is a function, reverse to f relatively variable φ (will mark that such function exists, as (φ, ψ, η) continuously differential, limit, positive determinate function. Functions $\Pi_j^{[n]}(\xi, \psi, \eta, t)$, $\bar{P}_j^{[n]}(\xi, \psi, \eta, t)$, $\bar{U}_j^{[n]}(\xi, \psi, \eta, t)$, $\breve{\Pi}_j^{[n]}\left(\breve{\xi}, \psi, \eta, t\right)$ $\breve{P}_j^{[n]}\left(\breve{\xi}, \psi, \eta, t\right)$, $\breve{U}_j^{[n]}\left(\breve{\xi}, \psi, \eta, t\right)$, $\tilde{\Pi}_j^{[n]}\left(\varphi, \tilde{\psi}, \eta, t\right)$, $\overline{\overline{\Pi}}_j^{[n]}\left(\varphi, \overline{\overline{\psi}}, \eta, t\right)$, $\widehat{\Pi}_j^{[n]}(\varphi, \psi, \tilde{\eta}, t)$, $\underline{\underline{\Pi}}_j^{[n]}\left(\varphi, \psi, \underline{\underline{\eta}}, t\right)$, $\tilde{P}_j\left(\varphi, \tilde{\psi}, \eta, t\right)$, $\overline{\overline{P}}_j\left(\varphi, \overline{\overline{\psi}}, \eta, t\right)$, $\widehat{P}_j(\varphi, \psi, \tilde{\eta}, t)$, $\underline{\underline{P}}_j\left(\varphi, \psi, \underline{\underline{\eta}}, t\right)$, $\tilde{U}_j\left(\varphi, \tilde{\psi}, \eta, t\right)$, $\overline{\overline{P}}_j\left(\varphi, \overline{\overline{\psi}}, \eta, t\right)$, $\widehat{P}_j^{[n]}(\varphi, \psi, \tilde{\eta}, t)$ are searched like [5]. The estimation of remaining members is conducted like [5].

4. THE NUMERICAL CALCULATION

We will bring results over of numerical experiment at $c_0^0(\varphi, \psi, \eta) = 100exp\left(-\varphi^2\right)$ mg/l, $c_*(\psi, \eta, t) = 100$ mg/l, $p_0^0(\varphi, \psi, \eta) = 0$ mg/l, $p_*(\psi, \eta, t) = 0$ mg/l, $u_0^0(\varphi, \psi, \eta) =$ 100 mg/l, $u_*(\psi, \eta, t) = 0$ mg/l, $\gamma = 1/10$, $\alpha = 1/4$, $\varepsilon = 0.01$. Considerable spatialness of filing up is characteristic, "monotony of narrowing" in direction from an entrance to the exit of filter for such filter (a practical worker "suggests" the choice of just the same form), and mutual orthogonal of verges along ribs and in angular points (it is essential for simplification of procedure of construction simply new conformal reflection). On basis [5] a calculative dynamic net was built in Gz: $\varphi(x, y, z) = \varphi_i \overset{\mathrm{df}}{=} \varphi_* + [(\varphi^* - \varphi_*)\, i]/n$, $i = \overline{0, n}$, ψ(x,y,z)$=$ $\bar{\psi}_j \overset{\mathrm{df}}{=} (Q_* j)/m$, $j = \overline{0, m}$, $\eta(x, y, z) = \bar{\eta}_k \overset{\mathrm{df}}{=} (Q^* k)/l$, $k = \overline{0, l}$ for $\varphi^* = 0$, $\varphi^* = 6000$, $\kappa = 1$, n = 30, m = 16, l = 16 (parameters of n, m and l were chosen from the condition of most similarity of the built net to cube), the filtration expense was found of Q = 0,341, the sizes of speed of filtration $|v|$ and functions $d_{io}(\varphi, \psi, \eta)$ (i = 1, 2) are calculated. As well the disclosures of the expected constructions do not exceed 0,001 [5]. On Figures 2-4 distributions, are represented c, p and u at t_0 =0,1, t_0 =0,3, t_0 =0,5, t_0 =0,7 h.

As you see concentration of admixtures and coagulants along filter-clarifier falls in course of time, that confirms a well-known fact. Distribution of concentration of flakes (see graph 3) at the beginning of filter grows headily, to the achievement of some state of "satiation", attaining that begins to fall. It is explained by that exactly in the first layers of filter under the action of coagulants, there is a reaction of creation the flakes.

5. CONCLUSIONS

The mathematical model of water treatment in clarifier is formed and analysed with taking into account influence of dose of reagent and irreversible coagulation of ad-

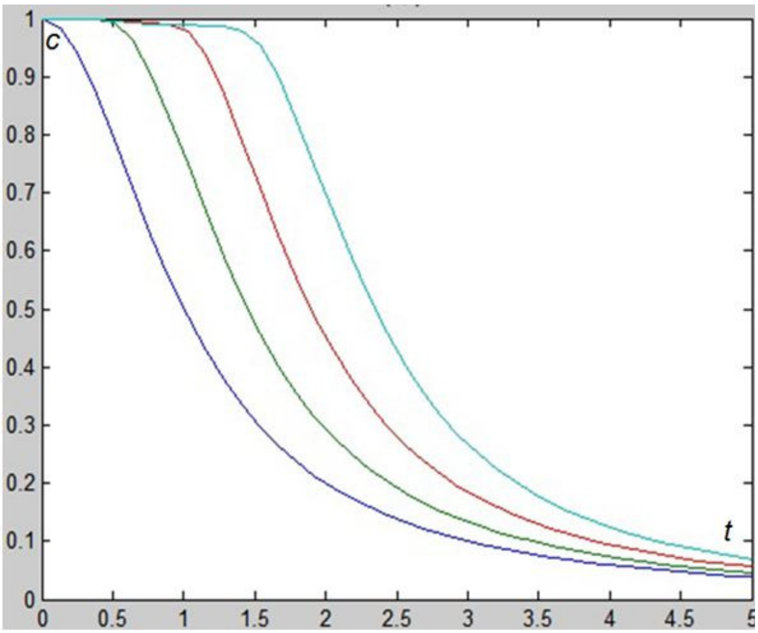

Figure 2: Distribution of concentration of admixtures c in different moments of time

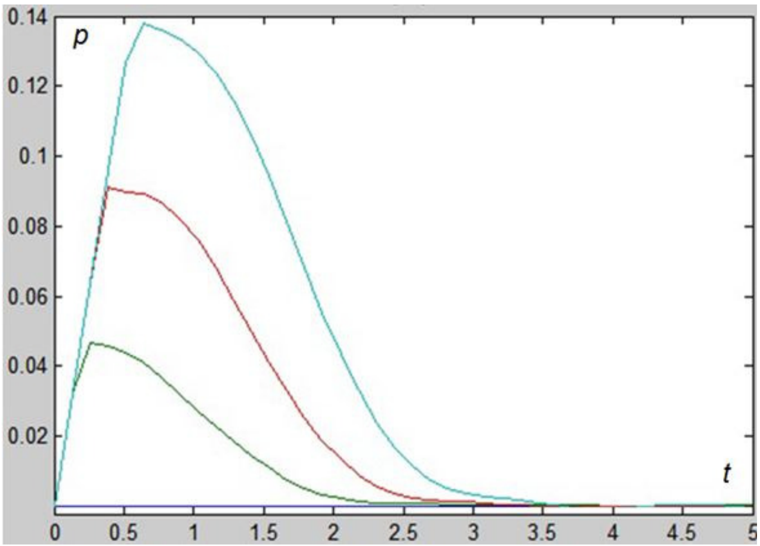

Figure 3: Distribution of concentration of flakes p in different moments of time

mixture particles. The algorithm of the numerical-asymptotic approaching of decision of corresponding model little nonlinear spatial problem is built for the system of differential equations as "convection-mass-transfer". Calculative dependences of concentrations of admixtures, flakes and substances are got for creation of flakes in filtration flow with the aim of engineering prognostication of dependence between production inputs of filter-clarifier and degree of efficiency of its work. For the future, generalization of the built model is envisaged with the aim of optimization of its basic

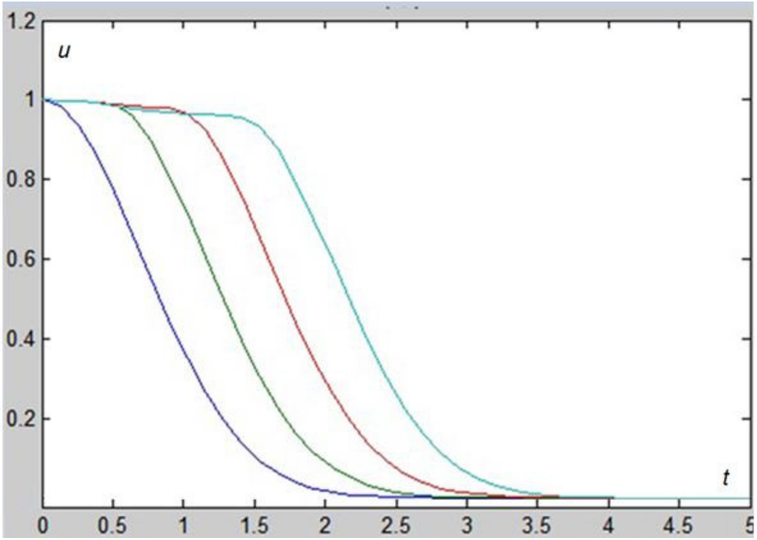

Figure 4: Distribution of concentration of substance for creation of flakes u in different moments of time

parameters.

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