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ON i-v FUZZY TRANSLATION OF i-v FUZZY BF-SUBALGEBRAS

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ABSTRACT: In this paper, the notion of an interval valued fuzzy translation of i-v fuzzy $BF-$subalgebra is introduced and we investigate some of their basic properties.

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1. INTRODUCTION

Zadeh [5], [6] extended of a fuzzy set into interval valued fuzzy set with an interval valued membership function. These concepts of fuzzy and interval valued fuzzy set are applied in various algebraic and topological structures by differet authors. Andrzej Walendziak [1] introduuced $BF-$akgebras in 2007. The authors of [4] and [7] studied Fuzzy $BF-$ algebras and interval valued fuzzy $BF-$algebras. In 2013 Chandramouleeswaran et al [3] dealt Fuzzy Translation and Fuzzy Multiplication in BF/BG-algebras and in 2015, Barbhuiya [2] focused the Fuzzy Translations and fuzzy multiplications of interval valued fuzzy BG-algebra. With all these motivations, in this paper, the notion on i-v fuzzy translation of i-v fuzzy $BF-$subalgebras has been discussed and some relevent results are also dealt.

2. PRELIMINARES

This section recalled some basic definitions needed for this work.

Definition 2.1. [1] A $BF-$algebra is a non-empty set X with a constant 0 and single binary operations $*$ satisfying the following axioms:

1. $x * x = 0,$

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2. $x * 0 = x$,

3. $0 * (x * y) = y * x \;\forall\; x, y, z \;\in\; X$.

Example 2.2. Let $X = \{0, 1, 2, 3, 4\}$ be a set which comprises the following table.

$*$	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then, $(X, *, 0)$ is BF-algebra.

Definition 2.3. [1] A non empty subset A of a $BF-$algebra $(X, +, -, 0)$ is called a $BF-$subalgebra of X, if $x * y \in A \quad \forall \; x, y \in X$

Definition 2.4. [4] Let μ be a fuzzy set in a $BF-$algebra X. Then μ is called a fuzzy β-subalgebra of X, if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\} \quad \forall \; x, y \in X$.

Definition 2.5. [6] An interval valued fuzzy set (briefly i-v fuzzy set) A defined on X is given by

$$A = \{(x, [\mu_A^L(x), \mu_A^U(x)])\} \quad \forall \; x \in X$$

(briefly denoted by $A = [\mu_A^L, \mu_A^U]$), where μ_A^L and μ_A^U are two fuzzy sets in X such that $\mu_A^L(x) \leq \mu_A^U(x) \;\; \forall \; x \in X$.

Let $\overline{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)] \quad \forall \; x \in X$ and let $D[0, 1]$ denotes the family of all closed subintervals of $[0, 1]$. If $\mu_A^L(x) = \mu_A^U(x) = c$, say, where $0 \leq c \leq 1$, then we have $\overline{\mu}_A(x) = [c, c]$ which we also assume, for the sake of convenience, to belong to $D[0, 1]$. Thus $\overline{\mu}_A(x) \in D[0, 1] \;\; \forall \; x \in X$, and therefore the i-v fuzzy set A is given by

$$A = \{(x, \overline{\mu}_A(x))\} \quad \forall \; x \in X,$$

where $\overline{\mu}_A : X \to D[0, 1]$.

Now let us define what is known as $refined\ mimimum$(briefly $rmim$) of two elements in $D[0, 1]$. We also define the symbols " $\geq$ " , " $\leq$ ", and " $=$ " in case of two elements in $D[0, 1]$.

Consider two elements $D_1 := [a_1, b_1]$ and $D_2 := [a_2, b_2] \in D[0, 1]$.

Then we have

$$\text{rmin}\,(D_1, D_2) = [\min\{a_1, a_2\}, \min\{b_1, b_2\}];$$

$D_1 \geq D_2$, if and only if $a_1 \geq a_2, \; b_1 \geq b_2$.

Similarly we may have $D_1 \leq D_2$ and $D_1 = D_2$.

Remark 2.6. Let $D_1 := [a_1, b_1]$ and $D_2 := [a_2, b_2] \in D[0, 1]$. Then:

1. $D_1 \leq D_2 \Leftrightarrow a_1 \leq a_2 \;\;\&\;\; b_1 \leq b_2$;
2. $D_1 = D_2 \Leftrightarrow a_1 = a_2 \;\;\&\;\; b_1 = b_2$;
3. $D_1 + D_2 = [a_1 + a_2, b_1 + b_2]$ whenever $a_1 + a_2 \leq 1$ and $b_1 + b_2 \leq 1$;
4. $D_1 - D_2 = [a_1 - a_2, b_1 - b_2]$ whenever $a_1 - a_2 \leq 1$ and $b_1 - b_2 \leq 1$.

Definition 2.7. [7] Let $\overline{\mu}_A$ be an i-v fuzzy subset in X. Then $\overline{\mu}_A$ is said to be interval valued fuzzy(i-v-fuzzy) $BF-$subalgebra of X, if

$$\overline{\mu}_A(x * y) \geq \text{rmin}\,\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}, \quad \forall\; x, y \in X.$$

Example 2.8. Consider the $BF-$algebra $X = \{0, a, b, c\}$ in Example 2.2.

Define an i-v fuzzy subset $\overline{\mu}$ of X defined by

$$\overline{\mu}(x) = \begin{cases} [0.6, 0.8], & x \neq 2, \\ [0.1, 0.2], & x = 2. \end{cases}$$

Then $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.

Definition 2.9. [1] Let μ be a fuzzy set of a $BF-$algebra X and $\alpha \in [0, T]$, where $T = 1 - sup\{\mu(x)/x \in X\}$. Then the fuzzy set $\mu^T_\alpha : X \to [0, 1]$ is called a fuzzy $\alpha-$translation of μ if $\mu^T_\alpha(x) = \mu(x) + \alpha$, $\forall x \in X$.

3. INTERVAL VALUED FUZZY TRANSLATIONS OF BF-SUBALGEBRA

This section deals with the notion of Interval valued fuzzy translation of interval valued fuzzy $BF-$subalgebra. In what follows, X denotes a $BF-$algebra and for any i-v fuzzy set $\overline{\mu}$ of X, we denote $\overline{T} = [1, 1] - \text{rsup}\,\{\overline{\mu}(x)/x \in X\}$ unless otherwise specified.

Definition 3.1. Let $\overline{\mu}$ be an i-v fuzzy set of X and $\overline{\alpha} \in [\overline{0}, \overline{T}]$, where $\overline{\alpha} = [\alpha^L, \alpha^U]$ with $\alpha^L \in [0, T^L]$ & $\alpha^U \in [0, T^U]$ and $\overline{0} = [0, 0]$. A mapping $\overline{\mu}^{\overline{T}}_{\overline{\alpha}} : X \to D[0, 1]$ is said to be an i-v fuzzy $\overline{\alpha}-$translation of $\overline{\mu}$ if it satisfies

$$\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x) = \overline{\mu}(x) + \overline{\alpha}, \quad \forall x \in X.$$

Example 3.2. Consider the $BF-$algebra X in example 2.2. Define an i-v fuzzy subset $\overline{\mu}$ of X by

$$\overline{\mu}(x) = \begin{cases} [0.4, 0.6], & x \neq 2, \\ [0.1, 0.3], & x = 2. \end{cases}$$

Then $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X. Here $\overline{T} = [1, 1] - \text{rsup}\,\{\overline{\mu}(x)/x \in X\} = [1, 1] - [0.4, 0.6] = [0.6, 0.4]$. Choose $\overline{\alpha} = [0.04, 0.08] \in [\overline{0}, \overline{T}]$. Then the i-v fuzzy set $\overline{\mu}^{\overline{T}}_{\overline{\alpha}} : X \to D[0, 1]$ is given by

$$\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x) = \begin{cases} [0.44, 0.68], & x \neq 2, \\ [0.14, 0.38], & x = 2 \end{cases}$$

is an i-v fuzzy $\overline{\alpha}-$translation of $\overline{\mu}$.

Theorem 3.3. *For any i-v fuzzy $BF-$subalgebra $\overline{\mu}$ of X and $\overline{\alpha} \in [\overline{0}, \overline{T}]$, the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x)$ of $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.*

Proof. Let $x, y \in X$ and $\overline{\alpha} \in [\overline{0}, \overline{T}]$. Then

$$\overline{\mu}(x * y) \geq \text{rmin}\,\{\overline{\mu}(x), \overline{\mu}(y)\}.$$

Hence

$$\begin{aligned} \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x * y) &= \overline{\mu}(x * y) + \overline{\alpha} \\ &\geq \text{rmin}\,\{\overline{\mu}(x), \overline{\mu}(y)\} + \overline{\alpha} \\ &= \text{rmin}\,\{\overline{\mu}(x) + \overline{\alpha}, \overline{\mu}(y) + \overline{\alpha}\} \\ &= \text{rmin}\,\{\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x), \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(y)\}. \end{aligned}$$

Therefore $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}$ of $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X. □

The following is the converse of the above theorem.

Theorem 3.4. *For any i-v fuzzy subset $\overline{\mu}$ of X and $\overline{\alpha} \in [\overline{0}, \overline{T}]$. If the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}$ of $\overline{\mu}$ is also an i-v fuzzy $BF-$subalgebra of X, then so is $\overline{\mu}$.*

Proof. Let $x, y \in X$. Assume that $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x)$ of $\overline{\mu}$ is a i-v fuzzy $BF-$subalgebra of X for some $\overline{\alpha} \in [\overline{0}, \overline{T}]$. Then

$$\begin{aligned} \overline{\mu}(x * y) + \overline{\alpha} &= \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x * y) \\ &\geq \text{rmin}\,\{\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x), \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(y)\} \\ &= \text{rmin}\,\{\overline{\mu}(x) + \overline{\alpha}, \overline{\mu}(y) + \overline{\alpha}\} \\ &= \text{rmin}\,\{\overline{\mu}(x), \overline{\mu}(y)\} + \overline{\alpha}, \\ \Rightarrow \overline{\mu}(x * y) &\geq \text{rmin}\,\{\overline{\mu}(x), \overline{\mu}(y)\}. \end{aligned}$$

Hence $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X. □

Remark 3.5. In general for any i-v fuzzy set $\overline{\mu}$ of X, the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(\overline{\alpha} \in [\overline{0}, \overline{T}])$ of $\overline{\mu}$ need not be an i-v fuzzy $BF-$subalgebra of X, as shown by the following example.

Let X be the $BF-$algebra given in Example 2.2. Consider the i-v fuzzy set $\overline{\mu}$:

$$\overline{\mu}(x) = \begin{cases} [0.5, 0.7], & x = 0, \\ [0.4, 0.6], & x = 1, \\ [0.3, 0.5], & x = 2, \\ [0.2, 0.4], & x = 3, \\ [0.1, 0.3], & x = 4. \end{cases}$$

Let $\overline{\alpha} = [0.02, 0.03]$. Then the corresponding $\overline{\alpha}-$translation is

$$\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x) = \begin{cases} [0.52, 0.73], & x = 0, \\ [0.42, 0.63], & x = 1, \\ [0.32, 0.53], & x = 2, \\ [0.22, 0.43], & x = 3, \\ [0.12, 0.33], & x = 4. \end{cases}$$

Hence, we have $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(2 * 3) = \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(4) = [0.12, 0.33] \not\geq [0.22, 0.43] = \text{rmin}\{\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(2), \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(3)\}$. Also $\overline{\mu}(2 * 3) = \overline{\mu}(4) = [0.1, 0.3] \not\geq [0.2, 0.4] = \text{rmin}\{\overline{\mu}(2), \overline{\mu}(3)\}$.

Therefore, both $\overline{\mu}$ and $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}$ are not i-v fuzzy $BF-$subalgebras of X.

Corollary 3.6. *Let $\overline{\mu}$ be an i-v fuzzy set of X. If $\overline{\alpha} = \overline{0}$ then the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}$ of $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.*

Theorem 3.7. *Let $\overline{\mu}$ be given an i-v fuzzy $BF-$subalgebra of X. Then for $\overline{\alpha}, \overline{\alpha'} \in [\overline{0}, \overline{T}]$, $(\overline{\mu}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}^{\overline{T}}_{\overline{\alpha'}})$ and $(\overline{\mu}^{\overline{T}}_{\overline{\alpha}} \cup \overline{\mu}^{\overline{T}}_{\overline{\alpha'}})$ are also an i-v fuzzy $BF-$subalgebra of X.*

Proof. Let $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}$ and $\overline{\mu}^{\overline{T}}_{\overline{\alpha'}}$ be two i-v fuzzy translation of an i-v fuzzy $BF-$subalgebra $\overline{\mu}$ of X, where $\overline{\alpha}, \overline{\alpha'} \in [\overline{0}, \overline{T}]$.

Assume that $\overline{\alpha} \leq \overline{\alpha'}$ by theorem 3.3 $\overline{\mu}^{\overline{T}}_{\overline{\alpha}}$ and $\overline{\mu}^{\overline{T}}_{\overline{\alpha'}}$ be two i-v fuzzy translation of $BF-$subalgebra of X. Now

$$\begin{aligned} (\overline{\mu}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}^{\overline{T}}_{\overline{\alpha'}})(x) &= \text{rmin}\{\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x), \overline{\mu}^{\overline{T}}_{\overline{\alpha'}}(x)\} \\ &= \text{rmin}\{\overline{\mu}(x) + \overline{\alpha}, \overline{\mu}(x) + \overline{\alpha'}\} \\ &= \overline{\mu}(x) + \overline{\alpha} \\ &= \overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x). \end{aligned}$$

Also

$$\begin{aligned}(\overline{\mu}^{\overline{T}}_{\overline{\alpha}} \cup \overline{\mu}^{\overline{T}}_{\overline{\alpha'}})(x) &= \text{rmax}\,\{\overline{\mu}^{\overline{T}}_{\overline{\alpha}}(x), \overline{\mu}^{\overline{T}}_{\overline{\alpha'}}(x)\} \\ &= \text{rmax}\,\{\overline{\mu}(x) + \overline{\alpha}, \overline{\mu}(x) + \overline{\alpha'}\} \\ &= \overline{\mu}(x) + \overline{\alpha'} \\ &= \overline{\mu}^{\overline{T}}_{\overline{\alpha'}}(x)\end{aligned}$$

$(\overline{\mu}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}^{\overline{T}}_{\overline{\alpha'}})$ and $(\overline{\mu}^{\overline{T}}_{\overline{\alpha}} \cup \overline{\mu}^{\overline{T}}_{\overline{\alpha'}})$ is an i-v fuzzy $BF-$subalgebra of X. □

Theorem 3.8. *Let $\overline{\mu}_1$ and $\overline{\mu}_2$ be two i-v fuzzy $BF-$subalgebras of X. Let $\overline{T}_{\mu_1} = rmin\{\overline{T}_{\mu_1}, \overline{T}_{\mu_2}\}$ where $\overline{T}_{\mu_1} = [1,1] - rsup\{\overline{\mu}_1(x) : x \in X\}$ and $\overline{T}_{\mu_2} = [1,1] - rsup\{\overline{\mu}_2(x) : x \in X\}$. Then the intersection of $\overline{\alpha}-$translation of $\overline{\mu}_1$ and $\overline{\alpha'}-$translation of $\overline{\mu_2}$ for some $\overline{\alpha}, \overline{\alpha'} \in [\overline{0}, \overline{T}]$ is an i-v fuzzy $BF-$subalgebra of X.*

Proof. Let $\overline{\mu}_1$ and $\overline{\mu}_2$ be two i-v fuzzy $BF-$subalgebra of X. Then by Theorem 3.3 $\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}}$ and $\overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}}$ are i-v fuzzy $BF-$subalgebra of X.

For $x, y \in X$:

$$\begin{aligned}(\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}})(x * y) &= \text{rmin}\,\{\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}}(x * y), \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}}(x * y)\} \\ &\geq \text{rmin}\,\{\text{rmin}\,\{\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}}(x), \overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha'}}(y)\}, \text{rmin}\,\{\overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}}(x), \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}}(y)\}\} \\ &= \text{rmin}\,\{\text{rmin}\,\{\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}}(x), \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}}(x)\}, \text{rmin}\,\{\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha'}}(y), \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}}(y)\}\} \\ &= \text{rmin}\,\{(\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}})(x), (\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}})(y)\}.\end{aligned}$$

Therefore $(\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}} \cap \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha'}})$ is an i-v fuzzy $BF-$subalgebra of X. □

Definition 3.9. Let $f : X \to Y$ be a function. Let $\overline{\mu}_X$ and $\overline{\mu}_Y$ be an i-v fuzzy $\overline{\alpha}-$translation on X and Y respectively. Then inverse image of $\overline{\mu}_Y$ under f is defined by $f^{-1}(\overline{\mu}_Y) = \{f^{-1}(\overline{\mu}_Y)^{\overline{T}}_{\overline{\alpha}}(x) : x \in X\}$ such that $f^{-1}(\overline{\mu}_Y)^{\overline{T}}_{\overline{\alpha}}(x) = \overline{\mu}_Y(f(x) + \overline{\alpha})$

Theorem 3.10. *Let X and Y be two $BF-$algebras and $f : X \to Y$ be a homomorphism. If the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}_Y$ of Y is an i-v fuzzy $BF-$subalgebra of Y, then $f^{-1}(\overline{\mu}_Y)$ is an i-v fuzzy $BF-$subalgebra of X.*

Proof. Let the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}_Y$ of Y be an i-v fuzzy $BF-$subalgebra of Y. Let $x, y \in Y$, then

$$\begin{aligned}f^{-1}(\overline{\mu}_Y{}^{\overline{T}}_{\overline{\alpha}})(x * y) &= f^{-1}(\overline{\mu}_Y)(x * y) + \overline{\alpha} \\ &= \overline{\mu}_Y(f(x * y) + \overline{\alpha}) \\ &= \overline{\mu}_Y(f(x) * f(y)) + \overline{\alpha} \\ &\geq \text{rmin}\,\{\overline{\mu}_Y(f(x) + \overline{\alpha}), \overline{\mu}_Y(f(y) + \overline{\alpha})\}\end{aligned}$$

$$= \text{rmin}\,\{f^{-1}(\overline{\mu}_Y{}^{\overline{T}}_{\overline{\alpha}})(x), f^{-1}(\overline{\mu}_Y{}^{\overline{T}}_{\overline{\alpha}})(y)\}.$$

Hence $f^{-1}(\overline{\mu}_Y)$ is an i-v fuzzy $BF-$subalgebra of X. □

Theorem 3.11. *Let X and Y be two $BF-$algebras and $f : X \to Y$ be a epimorphism. If the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}_X$ of X is an i-v fuzzy $BF-$subalgebra of X, then $f(\overline{\mu}_X)$ is an i-v fuzzy $BF-$subalgebra of Y.*

Proof. Let the i-v fuzzy $\overline{\alpha}-$translation $\overline{\mu}_X$ of X is an i-v fuzzy $BF-$subalgebra of X. Let $x, y \in Y$, then

$$\begin{aligned} f(\overline{\mu}_X{}^{\overline{T}}_{\overline{\alpha}})(x * y) &= f(\overline{\mu}_X)(x * y) + \overline{\alpha} \\ &= \overline{\mu}_X(f(x * y) + \overline{\alpha}) \\ &= \overline{\mu}_X(f(x) * f(y)) + \overline{\alpha} \\ &\geq \text{rmin}\,\{\overline{\mu}_X(f(x) + \overline{\alpha}), \overline{\mu}_X(f(y) + \overline{\alpha})\} \\ &= \text{rmin}\,\{f(\overline{\mu}_X{}^{\overline{T}}_{\overline{\alpha}})(x), f(\overline{\mu}_X{}^{\overline{T}}_{\overline{\alpha}})(y)\}. \end{aligned}$$

Hence $f(\overline{\mu}_X{}^{\overline{T}}_{\overline{\alpha}})$ is an i-v fuzzy $BF-$subalgebra of Y. □

Theorem 3.12. *Let $\overline{\mu}_1$ and $\overline{\mu}_2$ be two i-v fuzzy $BF-$subalgebras of X. Let $\overline{T} = rmin\,\{\overline{T}_{\overline{\mu}_1}, \overline{T}_{\overline{\mu}_2}\}$ where $\overline{T}_{\overline{\mu}_1} = [1, 1] - rsup\,\{\overline{\mu}_1(x) : x \in X\}$ and$\overline{T}_{\overline{\mu}_2} = [1, 1] - rsup\,\{\overline{\mu}_2(x) : x \in X\}$. Let $\overline{\alpha} \in [\overline{0}, \overline{T}]$. Then the $\overline{\alpha}-$translation of cartesian product $\overline{\mu}_1 \times \overline{\mu}_2$ of $\overline{\mu}_1$ and $\overline{\mu}_2$ is an i-v fuzzy $BF-$subalgebra of $X \times X$.*

Proof. Let$\overline{\mu}_1$ and $\overline{\mu}_2$ be an i-v fuzzy $BF-$subalgebra of a $BF-$algebra X and $\overline{\alpha} \in [\overline{0}, \overline{T}]$.

Now by Theorem 3.3, we obtain that $\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}}$ and $\overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha}}$ are i-v fuzzy $BF-$subalgebra of X.

Clearly $\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}} \times \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha}}$ is an i-v fuzzy $BF-$subalgebra of $X \times X$ and

$$\begin{aligned} (\overline{\mu}_1 \times \overline{\mu}_2)^{\overline{T}}_{\overline{\alpha}}(a, b) &= (\overline{\mu}_1 \times \overline{\mu}_2)(a, b) + \overline{\alpha} \\ &= \text{rmin}\,\{\overline{\mu}_1(a), \overline{\mu}_2(b)\} + \overline{\alpha} \\ &= \text{rmin}\,\{\overline{\mu}_1(a) + \overline{\alpha}, \overline{\mu}_2(b) + \overline{\alpha}\} \\ &= \text{rmin}\,\{\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}}(a), \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha}}(b)\} \\ &= (\overline{\mu}_1{}^{\overline{T}}_{\overline{\alpha}} \times \overline{\mu}_2{}^{\overline{T}}_{\overline{\alpha}})(a, b). \end{aligned}$$

Hence $(\overline{\mu}_1 \times \overline{\mu}_2)^{\overline{T}}_{\overline{\alpha}}$ is an i-v fuzzy $BF-$subalgebra of $X \times X$. □

4. INTERVAL VALUED FUZZY MULTIPLICATION

OF BF-SUBALGEBRA

This section introduce the notion of interval valued fuzzy $\overline{\alpha}-$multiplication. To illustrate the concept some examples are discussed and some simple results are proved.

Definition 4.1. Let $\overline{\mu}$ be an i-v fuzzy subset of X and $\overline{\phi} \in D[0,1]$. A mapping $\overline{\mu}^M_{\overline{\phi}} : X \to D[0,1]$ is said to be an i-v fuzzy $\overline{\phi}-$multiplication of $\overline{\mu}$ if it satisfies $\overline{\mu}^M_{\overline{\phi}}(x) = \overline{\phi}.\overline{\mu}(x) \ \ \forall\, x \in X$.

Example 4.2. Consider the i-v fuzzy set in example 3.2. Let $\overline{\phi} = [0.2, 0.3]$. Then the $\overline{\phi}-$multiplication of i-v fuzzy set $\overline{\mu}$ is given by

$$\overline{\mu}^M_{\overline{\phi}}(x) = \overline{\phi}.\overline{\mu}(x) = \begin{cases} [0.08, 0.18], & x \neq 2, \\ [0.02, 0.09], & x = 2. \end{cases}$$

Theorem 4.3. *For any i-v fuzzy $BF-$subalgebra $\overline{\mu}$ of X and $\overline{\phi} \in D[0,1]$, the i-v fuzzy $\overline{\phi}-$multiplication $\overline{\mu}^M_{\overline{\phi}}(x)$ of $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.*

Proof. Let $x, y \in X$ and $\overline{\phi} \in D[0,1]$, Then

$$\overline{\mu}(x * y) \geq \text{rmin}\,\{\overline{\mu}(x), \overline{\mu}(y)\}.$$

Now, we obtain

$$\begin{aligned} \overline{\mu}^M_{\overline{\phi}}(x * y) &= \overline{\phi}.\overline{\mu}(x * y) \\ &\geq \overline{\phi}.\text{rmin}\,\{\overline{\mu}(x), \overline{\mu}(y)\} \\ &= \text{rmin}\,\{\overline{\phi}.\overline{\mu}(x), \overline{\phi}.\overline{\mu}(y)\} \\ &= \text{rmin}\,\{\overline{\mu}^M_{\overline{\phi}}(x), \overline{\mu}^M_{\overline{\phi}}(y)\}. \end{aligned}$$

Hence $\overline{\mu}^M_{\overline{\phi}}$ of $\overline{\mu}$ is a i-v fuzzy $BF-$subalgebra of X. □

The following is the converse of the above theorem.

Theorem 4.4. *For any i-v fuzzy subset $\overline{\mu}$ of X and $\overline{\phi} \in D[0,1]$. If the i-v fuzzy $\overline{\phi}-$multiplication $\overline{\mu}^T_{\overline{\phi}}$ of $\overline{\mu}$ is also an i-v fuzzy $BF-$subalgebra of X, then so is $\overline{\mu}$.*

Proof. Let $x, y \in X$.

Assume that $\overline{\mu}^T_{\overline{\phi}}(x)$ of $\overline{\mu}$ is a i-v fuzzy $BF-$subalgebra of X for some $\overline{\phi} \in D[0,1]$. Then

$$\begin{aligned} \overline{\phi}.\overline{\mu}(x * y) &= \overline{\mu}^M_{\overline{\phi}}(x * y) \\ &\geq \text{rmin}\,\{\overline{\mu}^M_{\overline{\phi}}(x), \overline{\mu}^M_{\overline{\phi}}(y)\} \end{aligned}$$

$$= \text{rmin}\{\overline{\phi}.\overline{\mu}(x), \overline{\phi}.\overline{\mu}(y)\}$$
$$= \overline{\phi}.\text{rmin}\{\overline{\mu}(x), \overline{\mu}(y)\}.$$

Therefore

$$\overline{\mu}(x * y) \geq \text{rmin}\{\overline{\mu}(x), \overline{\mu}(y)\}.$$

Hence $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X. □

Definition 4.5. Let $\overline{\mu}$ be an i-v fuzzy subset of X, $\overline{\phi} \in D[0,1]$ and $\overline{\alpha} \in [\overline{0}, \overline{T}]$. A mapping $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}} : X \to D[0,1]$ is said to be an i-v fuzzy magnified $-\overline{\phi}\overline{\alpha}-$translation of $\overline{\mu}$ if it satisfies $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x) = \overline{\phi}.\overline{\mu}(x) + \overline{\alpha}\ \ \forall\, x \in X$.

Example 4.6. Consider the $BF-$algebra Xin example 2.2. Define an i-v fuzzy subset $\overline{\mu}$ of X by

$$\overline{\mu}(x) = \begin{cases} [0.4, 0.6], & x \neq 2, \\ [0.1, 0.3], & x = 2. \end{cases}$$

Then $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X. Here $\overline{T} = [1,1] - \text{rsup}\{\overline{\mu}(x)/x \in X\} = [1,1] - [0.4, 0.6] = [0.6, 0.4]$. Choose $\overline{\alpha} = [0.04, 0.08] \in [[0,0], [0.6, 0.4]$ and $\overline{\phi} = [0.1, 0.3] \in D[0,1]$. Then the i-v fuzzy set $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}} : X \to D[0,1]$ is given by

$$\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x) = \begin{cases} [0.08, 0.26], & x \neq 2, \\ [0.05, 0.17], & x = 2 \end{cases}$$

is an i-v fuzzy magnified $-\overline{\phi}\overline{\alpha}-$translation of $\overline{\mu}$.

Theorem 4.7. *Let $\overline{\mu}$ be an i-v fuzzy subset of X, $\overline{\phi} \in D[0,1]$ and $\overline{\alpha} \in [\overline{0}, \overline{T}]$. A mapping $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}} : X \to D[0,1]$ is an i-v fuzzy magnified-$\overline{\phi}\overline{\alpha}$-translation of $\overline{\mu}$. Then $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X if and only if $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}$ is an i-v fuzzy $BF-$subalgebra of X.*

Proof. Let $\overline{\mu}$ be an i-v fuzzy subset of X, $\overline{\phi} \in D[0,1]$ and $\overline{\alpha} \in [\overline{0}, \overline{T}]$. A mapping $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}} : X \to D[0,1]$ is said to be an i-v fuzzy magnified-$\overline{\phi}\overline{\alpha}$-translation of $\overline{\mu}$.

Assume that $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.

Then $\overline{\mu}(x * y) \geq \text{rmin}\{\overline{\mu}(x), \overline{\mu}(y)\}$. Now

$$\begin{aligned}
\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x * y) &= \overline{\phi}.\overline{\mu}(x * y) + \overline{\alpha} \\
&\geq \overline{\phi}.\text{rmin}\{\overline{\mu}(x), \overline{\mu}(y)\} + \overline{\alpha} \\
&= \text{rmin}\{\overline{\phi}.\overline{\mu}(x) + \overline{\alpha}, \overline{\phi}.\overline{\mu}(y) + \overline{\alpha}\} \\
&= \text{rmin}\{\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x), \overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(y)\}.
\end{aligned}$$

So, $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}$ of $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.

Assume that $\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x)$ of $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X.
Hence

$$\begin{aligned}\overline{\phi}.\overline{\mu}(x*y)+\overline{\alpha} &= \overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x*y)\\ &\geq \text{rmin}\,\{\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(x),\overline{\mu}^{M\overline{T}}_{\overline{\phi}\overline{\alpha}}(y)\}\\ &= \text{rmin}\,\{\overline{\phi}.\overline{\mu}(x)+\overline{\alpha},\overline{\phi}.\overline{\mu}(y)+\overline{\alpha}\}\\ &= \overline{\phi}.\text{rmin}\,\{\overline{\mu}(x),\overline{\mu}(y)\}+\overline{\alpha}\end{aligned}$$

Therefore $\overline{\mu}$ is an i-v fuzzy $BF-$subalgebra of X. □

5. CONCLUSION

An investigation on the i-v fuzzy translation and i-v fuzzy multiplication of BF-algebras are done and several interesting results are observed. In the future work it can be extended for intuitionistic i-v fuzzy and cubic fuzzy settings on BF-algebras.

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