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NON-INSTANTANEOUS DETERIORATING ITEMS WITH POWER DEMAND RATE AND SHORTAGES UNDER PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT: In this paper, we propose an appropriate inventory model for non instantaneous deteriorating items over power demand rate with permissible delay in payments and time dependent holding cost.In this model, the shortages are allowed under fully backlogged condition. We consider that the items are deteriorated with respect to time.To illustrate the optimal solutions by finding an optimal cycle time with the necessary and enough conditions for the existence and uniqueness of the optimal solutions. Finally, we demonstrate the numerical instance and sensitivity of the proposed model and get the optimal solution by using the tool of Matlab.

AMS Subject Classification: 90B05

1. INTRODUCTION

Generally refers to an accumulation over time of work waiting to be done or orders to be fulfilled is known as backlogging. In manufacturing industries, backlog refers to unfinished work or to customer orders that have been received but are either incomplete or in the process of completion.

In real life situation, since there is inventory loss by deterioration, controlling and maintaining inventories of deteriorating items becomes an important problem for decision makers in modern organization. Deterioration refers to damage, breakage, spoilage, dryness, vaporization, etc. of the products. Also Non-instantaneous deteri-

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oration means many goods maintain freshness or original condition for a particular period of time. During this period deterioration would not take place.

Trade credit means the delay in the payment offered by the supplier is a kind of price discount since paying later indirectly reduces the cost of holding, and it encourages the retailer to increase their order quantity.The classical economic order quantity model assumes that the retailer must pay the amount for the purchased items as soon as the items are received. However in practice the supplier may offer the retailer a delay period known as the trade credit period to settle his account within the fixed permitted settlement period. Offering such a credit period to the retailer will encourage the supplier?s selling and reduce on-hand stock level. Simultaneously, without a primary payment the retailer can take the advantages of a credit period to reduce cost and increase profit. Thus the delay in the payment offered by the supplier is a kind of price-discount since paying later indirectly reduces the cost of holding and it encourages the retailer to increase their order quantity. Moreover, during this credit period the retailer can start to accumulate revenues on the sales and earn interest on that revenue. Hence, paying later indirectly reduces the cost of holding stock. But a higher interest is charged if the payment is not settled by end of this credit period. Hence trade credit can play an important role in inventory model for both the suppliers as well as the retailers.

Horng and Wen, in [5], derived a partial backlogging inventory model for non-instantaneous deteriorating items with stock-dependent demand rate under inflation over a finite planning horizon. In [9], Palanivel and Uthayakumar Power pattern demand, Weibull two parameter deterioration rate and holding cost is expressed as linearly increasing functions of time are considered in this model. Also the model is very practical for the industries in which the holding cost is depending upon the time. In [10], Shalu considered and delay in payments is permitted. Shortages are allowed and assumed completely backlogged. In [8] Maryam et al.developed an economic ordering policy model for non-instantaneous deteriorating items with selling price- and inflation-induced demand under the effect of inflation, permissible delay in payments and customer returns. In [7] Kun et al considered a problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. Also analytically identify the best circumstance among these special cases based on the minimum total relevant cost per unit time. In [13] Vinod assumed to optimizing the total inventory cost for the business enterprises where holding cost and deterioration rate both are time dependent and salvage value is incorporated to the deteriorated items. In [1], Annadurai and Uthayakumar formulated an inventory model under two-levels of credit policy for deteriorating items by assuming the demand is a function of credit period offered by the retailer to the customers. Annadurai and Uthayakumar (see [2]) determined the retailers optimal ordering policy by finding the optimal length of inventory interval with positive inventory and the optimal length of order cycle for minimizing the cost. Vandana and Sharma (see [12]) developed an inventory model for deteriorating items with nonlinear demand rate, under the condition of permissible delay in payments, where the suppliers provided permissible delay in payments to the retailers. In [3]investigated the effects of initial inspection, interest earned on selling price and interest earned after fulfilling the back orders on the retailer ordering policy. In [6] Khanra developed for a deteriorating item having time dependent demand when delay in payment is permissible.

Hesham and Ahmed in [4] presented with a selling price-dependent demand rate, a storage time-dependent holding cost, and an order size-dependent purchase cost based on all-units quantity discount. Also mathematical model is constructed, and a solution methodology is developed for determining the optimal solution. [11] Sunil and Pravin considered for deterioration with time varying holding cost, shortage are permitted and kept backlogged. Vinod et al in [14] considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deterioration is time proportional. Also solved analytically by minimizing the total inventory cost. The result is illustrated with numerical example for the model.

2. NOTATIONS AND ASSUMPTIONS

2.1. NOTATIONS

The notations are used in the paper are listed as follows:

p_1	Purchasing cost per unit
h	Holding cost per unit per unit time excluding the capital cost
s	Shortage cost for backlogged items per unit per year
p	Selling price per unit per year
t_d	length of time per unit per year in which the product exhibits no deterioration
t_1	length of time in which there is no inventory shortage$(t_1 > t_d)$
T	duration of the replenishment cycle$(T > t_1)$
Q	order quantity per unit per year
t_1^*	optimal length of time in which there is no inventory shortage
I_0	maximum inventory level
I_e	interest earned per dollar

T_p	interest charged per dollar
$I_1(t)$	inventory level at time $t \in [0, t_d]$
$I_2(t)$	inventory level at time $t \in [t_d, t_1]$
$I_3(t)$	inventory level at time $t \in [t_1, T]$
M	trade credit period per unit per year
$TC(t_1)$	total minimum relevant cost for the inventory system

2.2. ASSUMPTIONS

1. The inventory system involves a single type of items.
2. Replenishment rate is infinite and replenishment size is constant.
3. The lead time is zero.
4. T is the fixed length of each production cycle.
5. The deterioration rate function $\theta(t)$ is considered as a time dependent deterioration rate defined as $\theta(t) = \alpha(t - t_d)$, for $t > 0$ and $0 < \alpha << 1$.
6. The time dependent holding cost $h(t) = x + yt$ where $x > 0$ and $y \neq 0$.
7. t_d is constant and $t_d < t_1$.
8. During the trade credit period M the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pay off units bought and starts to pay off the capital opportunity cost.

3. PROPOSED MODEL

The inventory system evolves as follows, I_0 units of item arrive at the inventory system at the beginning of each cycle. During the time interval $[0, t_d]$ the inventory level is decreasing only owing to demand rate. The inventory level is dropping to zero due to demand rate and deterioration during the time interval $[t_d, t_1]$. Then the shortage interval keeps to the endof the current order cycle.Finally, the shortages occur due to demand and completely backlogged during the time interval $[t_1, T]$.

4. MATHEMATICAL FORMULATION

The inventory level, decreases only owing to demand rate during the time interval $[0, t_d]$. Hence, the differential equation representing the inventory status is

$$\frac{dI_1(t)}{dt} = -ab^n, 0 \leq t \leq t_d, \text{ where } a \text{ and } b \text{ are integers and } n = 0, 1, 2, 3.... \quad (1)$$

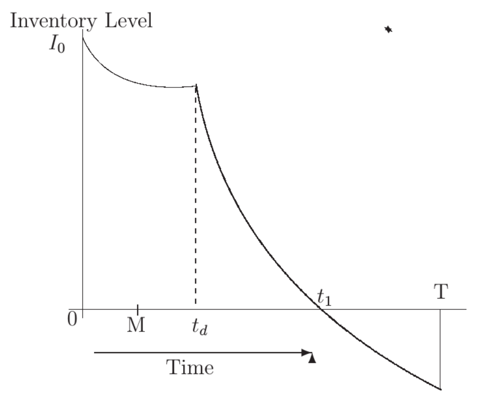

Figure 1: Graphical representation of inventory system $0 < M < t_d$

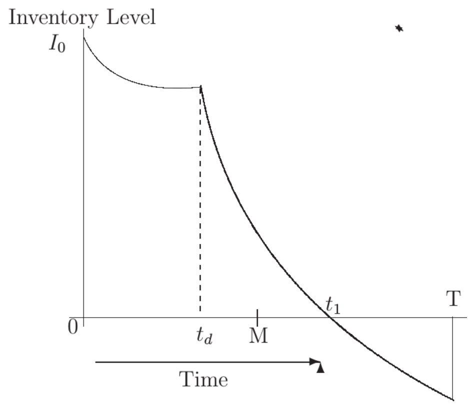

Figure 2: Graphical representation of inventory system $t_d < M < t_1$

In the time interval $[t_d, t_1]$ the inventory level decreases due to demand and deterioration

$$\frac{dI_2(t)}{dt} + \alpha(t - t_d)I_2(t) = -ab^n, t_d \leq t \leq t_1,$$

where a and b are integers and n=0,1,2,3... (2)

and during time interval $[t_1, T]$ inventory level decreases due to complete backlogging is given as below

$$\frac{dI_3(t)}{dt} = -ab^n, t_1 \leq t \leq T, \text{ where a and b are integers and n=0,1,2,3...} \quad (3)$$

Since, higher values of α are very small. So, we ignore the higher powers of α. Now, we solve the differential equations (1), (2), and (3) by using the boundary conditions $I_1(0) = I_0$,$I_2(t_1) = 0$ and $I_3(t_1) = 0$. Thus we have,

$$I_1(t) = -ab^n. \quad (4)$$

To solving (2), first, we expand the exponent terms by Taylors series expansions, we get

$$e^{\alpha t(\frac{t^2}{2} - t_d)} = 1 + (\alpha t(\frac{t^2}{2} - t_d)) + \frac{1}{2}(\alpha t(\frac{t^2}{2} - t_d)) + \dots .$$

Neglecting the highest power of α, we get

$$e^{\alpha t(\frac{t^2}{2} - t_d)} = 1 + (\alpha t(\frac{t^2}{2} - t_d)).$$

Now solve the linear differential equation. First, calculate the integrating factor

$$I.F = 1 + (\alpha t(\frac{t}{2} - t_d)), \text{ by our assumption } \alpha \ll 1.$$

After that,solving the above differential equation, we get

$$I_2(t) = ab^n(1-\alpha t(\frac{t}{2}-t_d))\left[\left(t_1-t\right)+\frac{\alpha}{6}\left(t_1^3-t^3\right)-\frac{\alpha t_d}{2}\left(t_1^2-t^2\right)\right], \tag{5}$$

$$I_3(t) = -ab^n(t), t_1 \le t \le T. \tag{6}$$

Considering the continuity of $I(t)$ at $t = t_d$ one can find(4) and (5) as

$$\begin{aligned} I_1(t_d) =& -ab^n t_d + I_0, \\ I_2(t_d) =& ab^n\left(1+\alpha t_d\left(\frac{t_d}{2}\right)\right)\left(\left(t_1-t_d\right)+\frac{\alpha}{6}\left(t_1^3-t_d^3\right)-\frac{\alpha t_d}{2}\left(t_1^2-t_d^2\right)\right), \\ I_0 =& ab^n\left(1+\alpha t_d\left(\frac{t_d}{2}\right)\right)\left(\left(t_1-t_d\right)+\frac{\alpha}{6}\left(t_1^3-t_d^3\right)-\frac{\alpha t_d}{2}\left(t_1^2-t_d^2\right)\right) \\ &+ ab^n t_d. \end{aligned} \tag{7}$$

Letting t=T in (6) then we obtain the maximum amount of demand, which is completely backlogged per cycle

$$\begin{aligned} X =& -I_3(T) \\ =& ab^n T. \end{aligned} \tag{8}$$

Then the total order quantity (Q) per unit per cycle is

$$\begin{aligned} Q =& I_0 + X, \\ Q =& ab^n\left(1+\alpha t_d\left(\frac{t_d}{2}\right)\right)\left(\left(t_1-t_d\right)+\frac{\alpha}{6}\left(t_1^3-t_d^3\right)-\frac{\alpha t_d}{2}\left(t_1^2-t_d^2\right)\right) \\ &+ ab^n t_d + ab^n T. \end{aligned} \tag{9}$$

Below, we calculate the inventory costs per cycle, which consists the following costs:

(i) Ordering Cost Ordering Cost = A.

(ii) Holding Cost (HC)

$$
\begin{aligned}
HC = xab^n\Bigg(&-\frac{t_d^2}{2}+\bigg(1+\frac{\alpha t_d^2}{2}\bigg)\Bigg(\bigg(t_1t_d-t_d^2\bigg)+\frac{\alpha}{6}\bigg(t_1^3t_d-t_d^4\bigg)-\frac{\alpha t_d}{2}\bigg(t_1^2t_d\\
&-t_d^2\bigg)\Bigg)+t_d^2\Bigg)+yab^n\Bigg[-\frac{t_d^2}{3}+\bigg(1+\frac{\alpha t_d^2}{2}\bigg)\Bigg(\bigg(\frac{t_1t_d^2}{2}-\frac{t_d^2}{2}\bigg)+\frac{\alpha}{6}\bigg(\frac{t_1^3t_d^5}{2}-\frac{t_d^5}{2}\bigg)\\
&-\frac{\alpha t_d}{2}\bigg(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{2}\bigg)\Bigg)+\frac{t_d^3}{2}\Bigg]+xab^n\Bigg(\Bigg[\frac{t_1^2}{2}+\frac{\alpha}{6}\bigg(\frac{3t_1^4}{4}\bigg)-\frac{\alpha t_d}{2}\bigg(\frac{2t_1^3}{3}\bigg)\Bigg]-\frac{\alpha}{2}\Bigg[\frac{t_1^4}{12}\\
&+\frac{\alpha}{6}\bigg(\frac{t_1^6}{6}\bigg)-\frac{\alpha t_d}{2}\bigg(\frac{2t_1^5}{15}\bigg)\Bigg]+\alpha t_d\Bigg[\frac{t_1^2}{2}+\frac{\alpha}{6}\bigg(\frac{3t_1^4}{4}\bigg)-\frac{\alpha t_d}{2}\bigg(\frac{2t_1^3}{3}\bigg)\Bigg]-\Bigg[\bigg(t_1t_d\\
&-\frac{t_d^2}{2}\bigg)+\frac{\alpha}{6}\bigg(t_1^3t_d-\frac{t_d^4}{4}\bigg)-\frac{\alpha t_d}{2}\bigg(t_1^2t_d-\frac{t_d^3}{3}\bigg)\Bigg]-\frac{\alpha}{2}\Bigg[\bigg(\frac{t_1t_d^3}{3}-\frac{t_d^2}{2}\bigg)+\frac{\alpha}{6}\bigg(\frac{t_1^3t_d^3}{3}\\
&-\frac{t_d^6}{6}\bigg)-\frac{\alpha t_d}{2}\bigg(\frac{t_1^2t_d^2}{3}-\frac{t_d^5}{5}\bigg)\Bigg]\Bigg)+yab^n\Bigg[\Bigg(\bigg(\frac{t_1^3}{6}+\frac{\alpha}{6}\bigg(\frac{3t_1^5}{10}\bigg)-\bigg(\frac{\alpha t_d}{2}\bigg(\frac{t_1^4}{4}\bigg)\bigg)\bigg)\Bigg)\\
&-\frac{\alpha}{2}\bigg(\frac{t_1^5}{5}+\frac{\alpha t_1^7}{56}-\frac{\alpha t_d}{2}\bigg(\frac{t_1^6}{12}\bigg)\bigg)+\alpha t_d\bigg(\bigg(\frac{t_1^3}{6}+\frac{\alpha t_1^5}{20}-\frac{\alpha t_d t_1^4}{8}\bigg)\bigg)-\Bigg(\Bigg[\frac{t_1t_d^2}{2}-\frac{t_d^3}{3}\\
&+\frac{\alpha}{6}\bigg(\frac{t_1^3t_d^2}{2}-\frac{t_d^5}{5}\bigg)-\frac{\alpha t_d}{2}\bigg(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{4}\bigg)\Bigg]-\frac{\alpha}{2}\Bigg[\bigg(\frac{t_1t_d^4}{4}-\frac{t_d^5}{5}+\frac{\alpha}{6}\bigg(\frac{t_1^3t_d^4}{4}-\frac{t_d^7}{7}\bigg)\\
&-\frac{\alpha t_d}{2}\bigg(\frac{t_1^2t_d^4}{4}-\frac{t_d^6}{6}\bigg)\bigg)\Bigg]+\alpha t_d\Bigg[\frac{t_1t_d^2}{2}-\frac{t_d^3}{2}+\frac{\alpha}{6}\bigg(\frac{t_1^3t_d^2}{2}-\frac{t_d^5}{5}-\frac{\alpha t_d}{2}\bigg(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{4}\bigg)\bigg)\Bigg]\Bigg)\Bigg].
\end{aligned}
\tag{10}
$$

(iii) The Shortage Cost due to Backlog (SC)

$$SC = sab^n\left[\frac{T^2}{2}-\frac{t_1^2}{2}\right].$$

(iv) The Deterioration Cost (DC)

$$DC = ab^n\left[p_1\left(\left(1+\frac{\alpha t_d^2}{2}\right)\left(t_1-t_d\right)+\frac{\alpha}{6}\left(t_1^3-t_d^3\right)-\frac{\alpha}{2}\left(t_1^2-t_d^2\right)-t_1+t_d\right)\right].$$

(v) Interest Payable. For each cycle, we need to consider the cases where the length of the credit period is longer or shorter than the length of time in which the product exhibits no deterioration (t_d) and the length of the period, with positive inventory of the item(t_1). So, we have three cases

$$I_p = \begin{cases} I_{p_1}, & 0 < M \le t_d; \\ I_{p_2}, & t_d < M \le t_1; \\ I_{p_3}, & t_1 < M \le T. \end{cases} \tag{11}$$

Case i: $0 < M \leq t_d$. In this case, payment for items is settled and retailer starts paying the capital opportunity cost for the items,

$$I_{p_1} = p_1 I_p \Bigg[- ab^n \left(\frac{t_d^2}{2} + \left(1 + \frac{\alpha t_d}{2}\right)\left(\left(t_1 t_d - t_d^2\right) + \frac{\alpha}{6}\left(t_1^3 t_d - t_d^4\right) - \frac{\alpha t_d}{2}\left(t_1^2 t_d - t_d^3\right)\right)\right.$$
$$\left. - \frac{M^2}{2} - \left(1 + \frac{\alpha t_d^2}{2}\right)\left(\left(t_1 M - t_d M\right) + \frac{\alpha}{6}\left(t_1^3 M - t_d^3 M\right) - \frac{\alpha t_d}{2}\left(t_1^2 M - t_d^2 M\right)\right)\right)$$
$$+ ab^n \Bigg[\Bigg[\left(\frac{t_1^2}{2} + \frac{\alpha t_1^4}{3} - \frac{\alpha t_d t_1^4}{3}\right) - \frac{\alpha}{2}\left(\frac{5t_1^4}{36} - \frac{2t_d t_1^4}{15}\right) + \alpha\left(\frac{t_d t_1^3}{6} + \frac{t_d t_1^5}{20} + \frac{t_d^2 t_1^4}{8}\right) - \Bigg[\Bigg(t_1 t_d$$
$$- \frac{t_d^2}{2}\Bigg) + \frac{\alpha}{6}\left(t_1^3 t_d - \frac{t_d^4}{4}\right) - \frac{\alpha t_d}{2}\left(t_1^2 t_d - \frac{t_d^3}{3}\right)\Bigg] - \frac{\alpha}{2}\Bigg[\left(\frac{t_1 t_d^2}{2} - \frac{t_d^4}{4} + \frac{t_1^3 t_d^3}{9} - \frac{t_d^6}{18}\right)$$
$$- t_d\left(\frac{t_1^2 t_d^3}{3} - \frac{t_d^5}{5}\right)\Bigg] + \alpha\Bigg[t_d\left(\left(\frac{t_1 t_d^2}{2} - \frac{t_d^3}{3}\right) + \frac{t_d}{6}\left(\frac{t_1^3 t_d^2}{2} - \frac{t_d^5}{5}\right) - \frac{t_d^2}{2}\left(\frac{t_1^2 t_d^2}{2} - \frac{t_d^4}{4}\right)\right)\Bigg]\Bigg]\Bigg]\Bigg].$$

Case ii: $t_d < M \leq t_1$. In this case interest payable is

$$I_{p_2} = p_1 I_p ab^n \Bigg[\Bigg[\frac{t_1^2}{2} + \frac{\alpha}{6}\left(\frac{3t_1^4}{4}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\Bigg] - \alpha\Bigg[\frac{t_1^4}{24} + \frac{\alpha}{6}\left(\frac{t_1^6}{12}\right)$$
$$- \frac{\alpha t_d}{2}\left(\frac{2t_1^5}{30}\right)\Bigg] + \alpha t_d\Bigg[\frac{t_1^2}{2} + \frac{\alpha}{6}\left(\frac{3t_1}{4}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\Bigg]$$
$$\Bigg[\left(\left(t_1 M - \frac{M^2}{2}\right) + \frac{\alpha}{6}\left(t_1^3 M - \frac{M^4}{4}\right) - \frac{\alpha t_d}{2}\left(t_1^2 M - \frac{M^3}{3}\right)\right)$$
$$- \alpha\left(\left(\frac{t_1 M^3}{6} - \frac{M^4}{8}\right) + \frac{\alpha}{6}\left(\frac{t_1^3 M^3}{6} - \frac{M^6}{12}\right) - \frac{\alpha t_d}{2}\left(\frac{t_1^2 M^3}{6} - \frac{M^5}{10}\right)\right)$$
$$+ \alpha t_d\left(\left(t_1 M - \frac{M^2}{2}\right) + \frac{\alpha}{6}\left(t_1^3 M - \frac{M^4}{4}\right) - \frac{\alpha t_d}{2}\left(t_1^2 M - \frac{M^3}{3}\right)\right)\Bigg]\Bigg] \tag{12}$$

Case iii: $t_1 < M \leq T$. There is no interest payable charged.

vi. Interest Earned. During the time, when the account is not settled, the retailer sells the goods and continues to accumulate sales revenue and earns the interest with rate I_e. Therefore, the interest earned per year is

$$I_e = \begin{cases} I_{e_1} & 0 < M \leq t_d \\ I_{e_2} & t_d < M \leq t_1 \\ I_{e_3} & t_1 < M \leq T \end{cases} \tag{13}$$

Case i: $0 < M \leq t_d$. The interest earned is

$$I_{e_1} = pI_e \left[\frac{ab^n M^2}{2}\right].$$

Case ii: $t_d < M \leq t_1$.

$$I_{e_2} = pI_e \left[\frac{ab^n M^2}{2}\right]$$

Case iii: $t_1 < M \leq T$.

$$I_{e_3} = pI_e \left[\frac{ab^n t_1^2}{2}\right] + \left(M - t_1\right)\left[ab^n t_1\right]$$

Therefore the total minimum relevant cost per unit time is denoted by TC

$$TC = \begin{cases} TC_1 & \frac{A+HC+SC+DC+I_{p_1}-I_{e_1}}{T} & 0 < M \leq t_d \\ TC_2 & \frac{A+HC+SC+DC+I_{p_2}-I_{e_2}}{T} & t_d < M \leq t_1 \\ TC_3 & \frac{A+HC+SC+DC+I_{p_3}-I_{e_3}}{T} & t_1 < M \leq T \end{cases} \tag{14}$$

5. SOLUTION PROCEDURE

Now, we will discuss the opportunity of each cases step by step. Since, TC_i for all $n = 1, 2, 3$ are continuous and well defined.

Case I. $0 < M \leq t_d$. To obtain the first order necessary condition for $TC_1(t_1)$ to be minimum, we differentiate $TC_1(t_1)$ with respect to t_1 and take the result equal to zero.

$$\frac{dTC_1}{dt_1} = 0 = \eta_1 \tag{15}$$

Thus, we find the value of t_{11} from (15)and differentiate with respect to t_1, we get

$$\frac{d^2TC_1}{dt^2} = \chi_1 \tag{16}$$

Since the expression of two derivative is highly nonlinear. Therefore, we are not writing the whole expression of χ_1. Thus, we have the following theorem.

Theorem 1.

1. *$\frac{dTC_1}{dt} = 0$, vanishes at $t_{11} = t_1^* \in [M, T)$, then $TC_1(t_1)$ not only exist, but unique and is minimum if $\chi_1 > 0$.*

2. *If $\chi_1 < 0$, then the $TC_1(t_1)$has its minimum at the point $t_{11} = t_1^* = M$.*

Proof. Based on the calculation of $\frac{dTC_1}{dt}$and χ_i for all $i = 1, 2$ the proof of Part (1)is obvious.

Now, we come to the proof of Part (2)is, If $\chi_1 \not> 0$, that means the value of t_1 for all $i = 1, 2$ is not a stationary value in $[M, \infty)$ i.e. $t_1 < M$.

Then the value of TC_i for all $i = 1, 2$ is monotonic increasing and monotonic decreasing function for $t_1 \in [M, \infty)$.

Now here $\lim_{t_1 \longrightarrow \infty} = \frac{dTC_i(t_1)}{dt_1} = +\infty$ for all $i = 1, 2$ will not have any stationary points $[M, \infty)$. Thus the value of $t_1$is unique.

Case 2. $(t_d < M \leq t_1)$. To obtain the first order necessary condition for $TC_2(t_1)$, is to be minimum, we differentiate $TC_2(t_1)$ with respect to t_1 and take the result equal to zero, we get

$$\frac{dTC_2(t_1)}{dt_1} = 0 = \eta_2 \tag{17}$$

Thus, we find the value of t_{12} from (17) and differentiate η_2 with respect to t_1

$$\frac{d\eta_2(t_1)}{dt_1} = \chi_2. \tag{18}$$

Since, the expression of two derivative is highly nonlinear.

Therefore, we are not writing the whole expression of χ_2. Thus, we have the following theorem.

Theorem 2.

1. *When χ_2=0, vanishes at $t_{12} = t_1^* \in [M, T)$, then $TC_2(t_1)$not only exist, but unique and is minimum if $\chi_2 > 0$.*

2. *If $\chi_2 < 0$, then the $TC_2(t_1)$has its minimum at the point $t_{12} = t_1^* = M$.*

Proof. Based on the calculation of $\frac{dTC_1}{dt}$and χ_i for all $i = 1, 2$ the proof of Part (1)is obvious.

Now, we come to the proof of Part (2)is, If $\chi_1 \not> 0$, that means the value of t_1 for all $i = 1, 2$ is not a stationary value in $[M, \infty)$ i.e. $t_1 < M$.

Then the value of TC_i for all $i = 1, 2$ is monotonic increasing and monotonic decreasing function for $t_1 \in [M, \infty)$.

Now here $\lim_{t_1 \longrightarrow \infty} = \frac{dTC_i(t_1)}{dt_1} = +\infty$ for all $i = 1, 2$ will not have any stationary points $[M, \infty)$. Thus the value of $t_1$is unique.

Case 3. $(t_1 < M \leq T)$. To obtain the first order necessary condition for $TC_3(t_1)$, is to be minimum, we differentiate $TC_3(t_1)$ with respect to t_1 and take the result equal to zero, we have

$$\frac{dTC_3(t_1)}{dt_1} = 0 = \eta_3 \tag{19}$$

Thus, we find the value of t_{13} from (19) with respect to t_1,i.e.$\frac{d\eta_3(t_1)}{dt_1} = \chi_3$, Since the expression of two derivative is highly nonlinear. Therefore, we are not writing the whole expression χ_3.

Now, we have the following Theorem

Theorem 3.

1. *When* η_3*=0, vanishes at* $t_{13} = t_1^* \in [0, M)$*, then* $TC_3(t_1)$*not only exist, but unique and is minimum if* $\chi_3 > 0$*.*

2. *If* $\chi_3 < 0$*, then the* $TC_3(t_1)$*has its minimum at the point* $t_{13} = t_1^* = M$*.*

Proof. The first part is obvious.

If, TC_3 does not have any constant value in $[0, M]$, then either TC_3 is monotonic increasing, or monotonic decreasing function of $t_1 \in [0, M]$.

Now, we differentiate TC_3 with respect to t_1 and take $t_1 \longrightarrow \infty$, thus, we get $\frac{dTC_3}{dt_1} \longrightarrow \infty$.

Thus our function is monotonic increasing function of $t_1 \in [0, M]$and TC_3 does not have any stationary value in $[0, M]$.

Hence, the maximum value of $t_1 = M$.

6. ALGORITHM

The procedure to find the optimal solution of t_1^*, is given as: **Step 1.** Find the minimum of $TC_1(t_1)$.

i Compute the η_1 and equate it, at 0 find t_1, compute χ_1, if $\chi_1 > 0$, then set $t_{11} = t_1^*$, otherwise go to the next step.

ii Set,$t_{11} = t_1^* = M$and find the value of $TC_1(t_1)$.

Step 2. Find the minimum of $TC_2(t_1)$.

i Compute the η_2 and equate it, at 0 find t_1, compute χ_2, if $\chi_2 > 0$, then set $t_{12} = t_1^*$, otherwise go to the next step.

ii Set,$t_{12} = t_1^* = M$and find the value of $TC_2(t_1)$.

Step 3. Find the minimum of $TC_3(t_1)$.

i Compute the η_3 and equate it, at 0 find t_1, compute χ_3, if $\chi_3 > 0$, then set $t_{13} = t_1^*$, otherwise go to the next step.

ii Set,$t_{13} = t_1^* = M$and find the value of $TC_3(t_1)$.

Step 4. To find the minimum $TC(t_1)$, we find

$$\min TC(t_1) = \min TC_1(t_1), TC_2(t_1), TC_3(t_1)$$

and accordingly select the optimal value of $t_1 = t_1^*$ and total relevant cost $TC(t_1)$.

Table 1: Numerical Example

M	t_1	t_d	Q	$TC1$
0.05	0.5351	0.385	937.5417	7462.9
M	t_1	t_d	Q	$TC2$
0.1223	0.5502	0.859	937.0743	7980.9
M	t_1	t_d	Q	TC
0.85	0.7065	—	937.2898	**6432.0**

7. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

We arbitrarily choose the value of a and b as \$25 per unit, \$15 per unit. Holding cost is x=\$10 per unit time , and y=\$5 per unit time, value of α=0.01, Shortage cost is s=\$30 per unit,ordering cost is A=\$250 per unit per order, interest payable rate is I_p=\$0.15 per year, interest earns by the retailer is I_e=\$0.12 per year, time there is no deterioration occurs t_d=0.068 per year, purchasing cost p_1=\$85 per unit, selling price of items are p=\$85 per unit, fixed cycle length is T=1.5 year.

Example 7.1 For Case i.$(M < t_d \leq t_1)$. Here, we consider $0 < M \leq t_d$ case, then we assume that M=0.05 and go to Step 1 of algorithm putting all the values of the parameters in the equation(15) and we find the value of χ_1, say $\chi_1 > 0$, then the optimal solution of $t_{11} = t_1^*$ not only exist, but also unique.

Example 7.2 For Case ii.$(t_d < M \leq t_1)$. In this case, we assume that M=0.1223 and go to Step 2 of algorithm putting all the values of the parameters in the equation(17) and we find the value of χ_2, say $\chi_2 > 0$, then the optimal solution of $t_{12} = t_1^*$ not only exist, but also unique.

Example 7.3 For Case iii.$(t_1 < M \leq T)$. In this case, we assume that M=0.85 and go to Step 3 of algorithm putting all the values of the parameters in the equation(19) and we find the value of χ_3, say $\chi_3 > 0$, then the optimal solution of $t_{13} = t_1^*$ not only exist, but also unique. Thus the optimal cycle time $t_1^* = t_{13} = 0.7065$ per year and the minimum total relevant cost is 6432.0.

8. OBSERVATIONS

The sensitivity analysis is performed by changing each of the parameters a, b, x, y, p_1, p and t_1.The result is presented in Table 2, Table 3 and Table 4. On the basis of results shown in Table 2, Table 3 and Table 4, we expose the following points as:

Table 2: Effect of Changes in the parameter of Example 1

Parameters	Values	Q	$TC1$
a	5	37.5017	458.8738
b	3		
x	8		
y	4		
p_1	65		
p	73		
t_1	0.641		
a	6	30.0013	399.0989
b	2		
x	7		
y	5		
p_1	63		
p	70		
t_1	0.532		
a	10	150.0067	1316.2
b	6		
x	4		
y	1		
p_1	70		
p	77		
t_1	0.621		
a	10	125.0056	1120.8
b	5		
x	5		
y	3		
p_1	78		
p	83		
t_1	0.452		
a	15	375.0167	3054.5
b	10		
x	6		
y	2		
p_1	75		
p	80		
t_1	0.762		

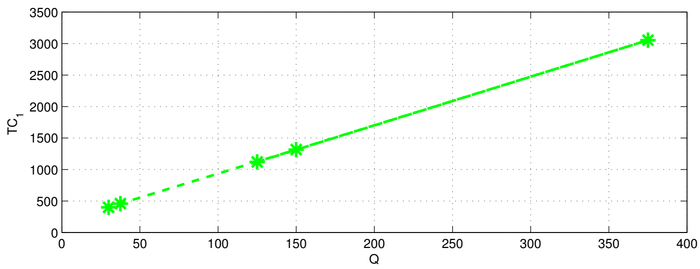

Figure 3: Total cost TC_1 with respect to t_1

Sensitivity analysis for Example 1. TC_1 has randomly increasing and decreasing if, we change the value of the parameters a, b, x, y, p_1, p, t_1. Thus a little changes in parameter is more affecting our model.

Table 3: Effect of Changes in the parameter of Example 2

Parameters	Values	Q	$TC2$
a	5		
b	3		
x	8		
y	4	37.4830	449.283
p_1	65		
p	73		
t_1	0.641		
a	6		
b	2		
x	7		
y	5	29.9864	409.8062
p_1	63		
p	70		
t_1	0.532		
a	10		
b	6		
x	4		
y	1	149.9319	1414.6
p_1	70		
p	77		
t_1	0.621		
a	10		
b	5		
x	5		
y	3	124.9432	1199.8
p_1	78		
p	83		
t_1	0.452		
a	15		
b	10		
x	6		
y	2	374.8297	3301.7
p_1	75		
p	80		
t_1	0.762		

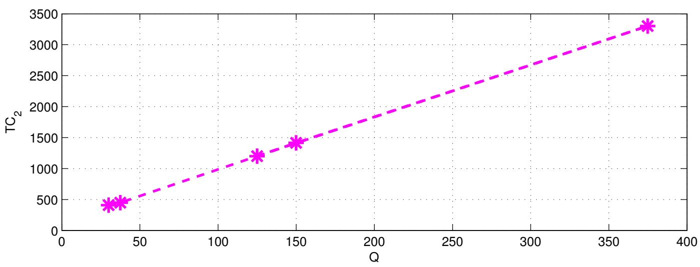

Figure 4: Total cost TC_2 with respect to t_1

Q has also increasing and decreasing if, we the change the value of the parameters a, b, x, y, p_1, p, t_1.

Sensitivity analysis for Example 2. TC_2 has randomly increasing and decreasing

Table 4: Effect of Changes in the parameter of Example 3

Parameters	Values	Q	$TC3$
a	5	37.4916	417.9854
b	3		
x	8		
y	4		
p_1	65		
p	73		
t_1	0.641		
a	6	29.9933	364.7148
b	2		
x	7		
y	5		
p_1	63		
p	70		
t_1	0.532		
a	10	150.0143	1157.1
b	6		
x	4		
y	1		
p_1	70		
p	77		
t_1	0.621		
a	10	124.9720	976.5814
b	5		
x	5		
y	3		
p_1	78		
p	83		
t_1	0.452		
a	15	375.0357	2681.2
b	10		
x	6		
y	2		
p_1	75		
p	80		
t_1	0.762		

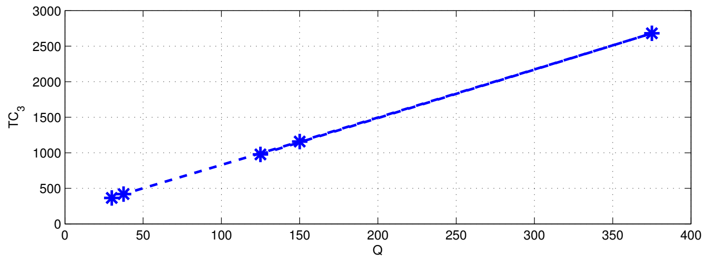

Figure 5: Total cost TC_3 with respect to t_1

if, we change the value of the parameters a, b, x, y, p_1, p, t_1. Thus a little changes in parameter is more affecting our model.

Q has also increasing and decreasing if, we the change the value of the parameters a, b, x, y, p_1, p, t_1.

Sensitivity analysis for Example 3. TC_3 has randomly increasing and decreasing if, we change the value of the parameters a, b, x, y, p_1, p, t_1. Thus a little changes in parameter is more affecting our model.

Q has also increasing and decreasing if, we the change the value of the parameters a, b, x, y, p_1, p, t_1.

9. MANAGERIAL IMPLICATIONS AND CONCLUSION

Based on the sensitivity analysis, as shown Table 2, Table 3 and Table 4, we can obtain the following managerial implications.

1. When the selling price p is increased, then the optimal shortage point has fixed value of Case 1, i.e. $M \leq t_d < t_1 < T$ and for Case 2, $t_d < M \leq t_1 < T$, while it increases in case 3, $t_d < t_1 < M \leq T$ is decreased.

2. When the selling price p is increased, then the total relevant cost for Case 1, i.e. $M \leq t_d < t_1 < T$ and for Case 2. $t_d < M \leq t_1 < T$, while it increases in case 3, $t_d < t_1 < M \leq T$ is decreased.

3. Our model is mainly applicable for those types of firms and factories, who is manufacturing the products like spare parts of new machineries,electronic components, fashionable commodities, etc.

In this paper we developed a non instantaneous deteriorating items with power demand rate and shortages under fully backlogged condition. Here suppliers provided the permissible delay in payments to the retailers. The present model is specially applicable for products like food items, electronic goods,etc., whose deterioration is non-instantaneous. The deterioration rate of items are not constant. Therefore, we consider that the items are deteriorating with respect to time.

The model developed in this paper can be enriched by extending more situations, such as multi-items,quantity discount policies,finite replenishment rate, Weibull distribution deterioration, time value money, etc.

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APPENDIX I

$$
\begin{aligned}
TC_1 = \frac{1}{T}\Bigg[& A + xab^n\Bigg(-\frac{t_d^2}{2} + \left(1+\frac{\alpha t_d^2}{2}\right)\Bigg(\left(t_1t_d - t_d^2\right) + \frac{\alpha}{6}\left(t_1^3 t_d - t_d^4\right) - \frac{\alpha t_d}{2}\Bigg(t_1^2 t_d \\
& - t_d^2\Bigg)\Bigg) + t_d^2\Bigg) + yab^n\Bigg[-\frac{t_d^2}{3} + \left(1+\frac{\alpha t_d^2}{2}\right)\Bigg(\left(\frac{t_1t_d^2}{2}-\frac{t_d^2}{2}\right)+\frac{\alpha}{6}\left(\frac{t_1^3t_d^5}{2}-\frac{t_d^5}{2}\right) \\
& -\frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{2}\right)\Bigg)+\frac{t_d^3}{2}\Bigg] + xab^n\Bigg(\Bigg[\frac{t_1^2}{2}+\frac{\alpha}{6}\left(\frac{3t_1^4}{4}\right)-\frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\Bigg]-\frac{\alpha}{2}\Bigg[\frac{t_1^4}{12} \\
& +\frac{\alpha}{6}\left(\frac{t_1^6}{6}\right)-\frac{\alpha t_d}{2}\left(\frac{2t_1^5}{15}\right)\Bigg]+\alpha t_d\Bigg[\frac{t_1^2}{2}+\frac{\alpha}{6}\left(\frac{3t_1^4}{4}\right)-\frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\Bigg]-\Bigg[\left(t_1t_d-\frac{t_d^2}{2}\right) \\
& +\frac{\alpha}{6}\left(t_1^3t_d-\frac{t_d^4}{4}\right)-\frac{\alpha t_d}{2}\left(t_1^2t_d-\frac{t_d^3}{3}\right)\Bigg]-\frac{\alpha}{2}\Bigg[\left(\frac{t_1t_d^3}{3}-\frac{t_d^2}{2}\right)+\frac{\alpha}{6}\left(\frac{t_1^3t_d^3}{3}-\frac{t_d^6}{6}\right) \\
& -\frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{3}-\frac{t_d^5}{5}\right)\Bigg]\Bigg)+yab^n\Bigg[\Bigg(\Bigg(\frac{t_1^3}{6}+\frac{\alpha}{6}\left(\frac{3t_1^5}{10}\right)-\left(\frac{\alpha t_d}{2}\left(\frac{t_1^4}{4}\right)\right)\Bigg)\Bigg)-\frac{\alpha}{2}\Bigg(\frac{t_1^5}{5}+\frac{\alpha t_1^7}{56} \\
& -\frac{\alpha t_d}{2}\left(\frac{t_1^6}{12}\right)\Bigg)+\alpha t_d\Bigg(\Bigg(\frac{t_1^3}{6}+\frac{\alpha t_1^5}{20}-\frac{\alpha t_d t_1^4}{8}\Bigg)\Bigg)-\Bigg(\Bigg[\frac{t_1t_d^2}{2}-\frac{t_d^3}{3}+\frac{\alpha}{6}\left(\frac{t_1^3t_d^2}{2}-\frac{t_d^5}{5}\right) \\
& -\frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{4}\right)\Bigg]-\frac{\alpha}{2}\Bigg[\Bigg(\frac{t_1t_d^4}{4}-\frac{t_d^5}{5}+\frac{\alpha}{6}\left(\frac{t_1^3t_d^4}{4}-\frac{t_d^7}{7}\right)-\frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^4}{4}-\frac{t_d^6}{6}\right)\Bigg)\Bigg] \\
& +\alpha t_d\Bigg[\frac{t_1t_d^2}{2}-\frac{t_d^3}{2}+\frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^2}{2}-\frac{t_d^5}{5}-\frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{4}\right)\Bigg)\Bigg]\Bigg)\Bigg]+sab^n\Bigg[\frac{T^2}{2}-\frac{t_1^2}{2}\Bigg] \\
& +ab^n\Bigg[p_1\Bigg(\left(1+\frac{\alpha t_d^2}{2}\right)\left(t_1-t_d\right)+\frac{\alpha}{6}\left(t_1^3-t_d^3\right)-\frac{\alpha}{2}\left(t_1^2-t_d^2\right)-t_1+t_d\Bigg)\Bigg]+p_1I_p\Bigg[\\
& -ab^n\Bigg(\frac{t_d^2}{2}+\left(1+\frac{\alpha t_d}{2}\right)\Bigg(\left(t_1t_d-t_d^2\right)+\frac{\alpha}{6}\left(t_1^3t_d-t_d^4\right)-\frac{\alpha t_d}{2}\left(t_1^2t_d-t_d^3\right)\Bigg)-\frac{M^2}{2} \\
& -\left(1+\frac{\alpha t_d^2}{2}\right)\Bigg(\left(t_1M-t_dM\right)+\frac{\alpha}{6}\left(t_1^3M-t_d^3M\right)-\frac{\alpha t_d}{2}\left(t_1^2M-t_d^2M\right)\Bigg)\Bigg) \\
& +ab^n\Bigg[\Bigg[\left(\frac{t_1^2}{2}+\frac{\alpha t_1^4}{3}-\frac{\alpha t_d t_1^4}{3}\right)-\frac{\alpha}{2}\left(\frac{5t_1^4}{36}-\frac{2t_dt_1^4}{15}\right)+\alpha\left(\frac{t_dt_1^3}{6}+\frac{t_dt_1^5}{20}+\frac{t_d^2t_1^4}{8}\right) \\
& -\Bigg[\left(t_1t_d-\frac{t_d^2}{2}\right)+\frac{\alpha}{6}\left(t_1^3t_d-\frac{t_d^4}{4}\right)-\frac{\alpha t_d}{2}\left(t_1^2t_d-\frac{t_d^3}{3}\right)\Bigg]-\frac{\alpha}{2}\Bigg[\Bigg(\frac{t_1t_d^2}{2}-\frac{t_d^4}{4}+\frac{t_1^3t_d^3}{9} \\
& -\frac{t_d^6}{18}\Bigg)-t_d\left(\frac{t_1^2t_d^3}{3}-\frac{t_d^5}{5}\right)\Bigg]+\alpha\Bigg[t_d\Bigg(\Bigg(\frac{t_1t_d^2}{2}-\frac{t_d^3}{3}\Bigg)+\frac{t_d}{6}\Bigg(\frac{t_1^3t_d^2}{2} \\
& -\frac{t_d^5}{5}\Bigg)-\frac{t_d^2}{2}\left(\frac{t_1^2t_d^2}{2}-\frac{t_d^4}{4}\right)\Bigg)\Bigg]\Bigg]\Bigg]\Bigg]\Bigg]-pI_e\left[\frac{ab^nM^2}{2}\right]\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
TC_2 = \frac{1}{T}\Bigg[& A + xab^n\Bigg(-\frac{t_d^2}{2} + \left(1+\frac{\alpha t_d^2}{2}\right)\Bigg(\left(\left(t_1t_d - t_d^2\right) + \frac{\alpha}{6}\left(t_1^3t_d - t_d^4\right) - \frac{\alpha t_d}{2}\Big(t_1^2t_d\right. \\
& \left.\left. - t_d^2\Big)\right)\right) + t_d^2\Bigg) + yab^n\Bigg[-\frac{t_d^2}{3} + \left(1+\frac{\alpha t_d^2}{2}\right)\Bigg(\left(\left(\frac{t_1t_d^2}{2} - \frac{t_d^2}{2}\right) + \frac{\alpha}{6}\left(\frac{t_1^3t_d^5}{2} - \frac{t_d^5}{2}\right)\right. \\
& \left. - \frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{2} - \frac{t_d^4}{2}\right)\right)\Bigg) + \frac{t_d^3}{2}\Bigg] + xab^n\Bigg(\left[\frac{t_1^2}{2} + \frac{\alpha}{6}\left(\frac{3t_1^4}{4}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\right] - \frac{\alpha}{2}\Bigg[\frac{t_1^4}{12} \\
& + \frac{\alpha}{6}\left(\frac{t_1^6}{6}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^5}{15}\right)\Bigg] + \alpha t_d\left[\frac{t_1^2}{2} + \frac{\alpha}{6}\left(\frac{3t_1^4}{4}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\right] - \Bigg[\Bigg(t_1t_d \\
& - \frac{t_d^2}{2}\Bigg) + \frac{\alpha}{6}\left(t_1^3t_d - \frac{t_d^4}{4}\right) - \frac{\alpha t_d}{2}\left(t_1^2t_d - \frac{t_d^3}{3}\right)\Bigg] - \frac{\alpha}{2}\Bigg[\left(\frac{t_1t_d^3}{3} - \frac{t_d^2}{2}\right) + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^3}{3} \\
& - \frac{t_d^6}{6}\Bigg) - \frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{3} - \frac{t_d^5}{5}\right)\Bigg]\Bigg) + yab^n\Bigg[\Bigg(\left(\frac{t_1^3}{6} + \frac{\alpha}{6}\left(\frac{3t_1^5}{10}\right) - \left(\frac{\alpha t_d}{2}\left(\frac{t_1^4}{4}\right)\right)\right) \\
& - \frac{\alpha}{2}\left(\frac{t_1^5}{5} + \frac{\alpha t_1^7}{56} - \frac{\alpha t_d}{2}\left(\frac{t_1^6}{12}\right)\right) + \alpha t_d\left(\left(\frac{t_1^3}{6} + \frac{\alpha t_1^5}{20} - \frac{\alpha t_d t_1^4}{8}\right)\right) - \Bigg(\Bigg[\frac{t_1t_d^2}{2} \\
& - \frac{t_d^3}{3} + \frac{\alpha}{6}\left(\frac{t_1^3t_d^2}{2} - \frac{t_d^5}{5}\right) - \frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^2}{2} - \frac{t_d^4}{4}\right)\Bigg] - \frac{\alpha}{2}\Bigg[\Bigg(\frac{t_1t_d^4}{4} - \frac{t_d^5}{5} + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^4}{4} \\
& - \frac{t_d^7}{7}\Bigg) - \frac{\alpha t_d}{2}\left(\frac{t_1^2t_d^4}{4} - \frac{t_d^6}{6}\right)\Bigg)\Bigg] + \alpha t_d\Bigg[\frac{t_1t_d^2}{2} - \frac{t_d^3}{2} + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^2}{2} - \frac{t_d^5}{5} - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^2t_d^2}{2} \\
& - \frac{t_d^4}{4}\Bigg)\Bigg)\Bigg]\Bigg)\Bigg] + sab^n\left[\frac{T^2}{2} - \frac{t_1^2}{2}\right] + ab^n\Bigg[p_1\Bigg(\left(1+\frac{\alpha t_d^2}{2}\right)\left(t_1 - t_d\right) + \frac{\alpha}{6}\left(t_1^3 - t_d^3\right) \\
& - \frac{\alpha}{2}\left(t_1^2 - t_d^2\right) - t_1 + t_d\Bigg)\Bigg] + p_1I_pab^n\Bigg[\left[\frac{t_1^2}{2} + \frac{\alpha}{6}\left(\frac{3t_1^4}{4}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\right] - \alpha\Bigg[\frac{t_1^4}{24} \\
& + \frac{\alpha}{6}\left(\frac{t_1^6}{12}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^5}{30}\right)\Bigg] + \alpha t_d\left[\frac{t_1^2}{2} + \frac{\alpha}{6}\left(\frac{3t_1}{4}\right) - \frac{\alpha t_d}{2}\left(\frac{2t_1^3}{3}\right)\right]\Bigg]\Bigg[\Bigg(\Bigg(t_1M \\
& - \frac{M^2}{2}\Bigg) + \frac{\alpha}{6}\left(t_1^3M - \frac{M^4}{4}\right) - \frac{\alpha t_d}{2}\left(t_1^2M - \frac{M^3}{3}\right)\Bigg) - \alpha\Bigg(\Bigg(\frac{t_1M^3}{6} \\
& - \frac{M^4}{8}\Bigg) + \frac{\alpha}{6}\left(\frac{t_1^3M^3}{6} - \frac{M^6}{12}\right) - \frac{\alpha t_d}{2}\left(\frac{t_1^2M^3}{6} - \frac{M^5}{10}\right)\Bigg) + \alpha t_d\Bigg(\Bigg(t_1M \\
& - \frac{M^2}{2}\Bigg) + \frac{\alpha}{6}\left(t_1^3M - \frac{M^4}{4}\right) - \frac{\alpha t_d}{2}\left(t_1^2M - \frac{M^3}{3}\right)\Bigg)\Bigg]\Bigg]pI_e\left[\frac{ab^nM^2}{2}\right]\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
TC_3 = \frac{1}{T}\Bigg[& A + xab^n\Bigg(-\frac{t_d^2}{2} + \Bigg(1+\frac{\alpha t_d^2}{2}\Bigg)\Bigg(\Bigg(t_1t_d - t_d^2\Bigg) + \frac{\alpha}{6}\Bigg(t_1^3t_d - t_d^4\Bigg) - \frac{\alpha t_d}{2}\Bigg(t_1^2 t_d \\
& - t_d^2\Bigg)\Bigg) + t_d^2\Bigg) + yab^n\Bigg[-\frac{t_d^2}{3} + \Bigg(1+\frac{\alpha t_d^2}{2}\Bigg)\Bigg(\Bigg(\frac{t_1t_d^2}{2} - \frac{t_d^2}{2}\Bigg) + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^5}{2} - \frac{t_d^5}{2}\Bigg) \\
& - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^2t_d^2}{2} - \frac{t_d^4}{2}\Bigg)\Bigg) + \frac{t_d^3}{2}\Bigg] + xab^n\Bigg(\Bigg[\frac{t_1^2}{2} + \frac{\alpha}{6}\Bigg(\frac{3t_1^4}{4}\Bigg) - \frac{\alpha t_d}{2}\Bigg(\frac{2t_1^3}{3}\Bigg)\Bigg] - \frac{\alpha}{2}\Bigg[\frac{t_1^4}{12} \\
& + \frac{\alpha}{6}\Bigg(\frac{t_1^6}{6}\Bigg) - \frac{\alpha t_d}{2}\Bigg(\frac{2t_1^5}{15}\Bigg)\Bigg] + \alpha t_d\Bigg[\frac{t_1^2}{2} + \frac{\alpha}{6}\Bigg(\frac{3t_1^4}{4}\Bigg) - \frac{\alpha t_d}{2}\Bigg(\frac{2t_1^3}{3}\Bigg)\Bigg] - \Bigg[\Bigg(t_1t_d \\
& - \frac{t_d^2}{2}\Bigg) + \frac{\alpha}{6}\Bigg(t_1^3t_d - \frac{t_d^4}{4}\Bigg) - \frac{\alpha t_d}{2}\Bigg(t_1^2t_d - \frac{t_d^3}{3}\Bigg)\Bigg] - \frac{\alpha}{2}\Bigg[\Bigg(\frac{t_1t_d^3}{3} - \frac{t_d^2}{2}\Bigg) + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^3}{3} \\
& - \frac{t_d^6}{6}\Bigg) - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^2t_d^2}{3} - \frac{t_d^5}{5}\Bigg)\Bigg]\Bigg]\Bigg) + yab^n\Bigg[\Bigg(\Bigg(\frac{t_1^3}{6} + \frac{\alpha}{6}\Bigg(\frac{3t_1^5}{10}\Bigg) - \Bigg(\frac{\alpha t_d}{2}\Bigg(\frac{t_1^4}{4}\Bigg)\Bigg)\Bigg) \\
& - \frac{\alpha}{2}\Bigg(\frac{t_1^5}{5} + \frac{\alpha t_1^7}{56} - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^6}{12}\Bigg)\Bigg) + \alpha t_d\Bigg(\Bigg(\frac{t_1^3}{6} + \frac{\alpha t_1^5}{20} - \frac{\alpha t_d t_1^4}{8}\Bigg)\Bigg) - \Bigg(\Bigg[\frac{t_1t_d^2}{2} \\
& - \frac{t_d^3}{3} + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^2}{2} - \frac{t_d^5}{5}\Bigg) - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^2t_d^2}{2} - \frac{t_d^4}{4}\Bigg)\Bigg] - \frac{\alpha}{2}\Bigg[\Bigg(\frac{t_1t_d^4}{4} - \frac{t_d^5}{5} + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^4}{4} \\
& - \frac{t_d^7}{7}\Bigg) - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^2t_d^4}{4} - \frac{t_d^6}{6}\Bigg)\Bigg)\Bigg)\Bigg] + \alpha t_d\Bigg[\frac{t_1t_d^2}{2} - \frac{t_d^3}{2} + \frac{\alpha}{6}\Bigg(\frac{t_1^3t_d^2}{2} - \frac{t_d^5}{5} - \frac{\alpha t_d}{2}\Bigg(\frac{t_1^2t_d^2}{2} \\
& - \frac{t_d^4}{4}\Bigg)\Bigg)\Bigg)\Bigg]\Bigg]\Bigg)\Bigg] + sab^n\Bigg[\frac{T^2}{2} - \frac{t_1^2}{2}\Bigg] + ab^n\Bigg[p_1\Bigg(\Bigg(1+\frac{\alpha t_d^2}{2}\Bigg)\Bigg(t_1 - t_d\Bigg) + \frac{\alpha}{6}\Bigg(t_1^3 \\
& - t_d^3\Bigg) - \frac{\alpha}{2}\Bigg(t_1^2 - t_d^2\Bigg) - t_1 + t_d\Bigg)\Bigg] - pI_e\Bigg[\frac{ab^nt_1^2}{2}\Bigg] + \Bigg(M - t_1\Bigg)\Bigg[ab^nt_1\Bigg]\Bigg].
\end{aligned}
$$

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