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LIMITED CAPACITY STOREHOUSE INVENTORY MODEL FOR DETERIORATING ITEMS WITH PRESERVATION TECHNOLOGY AND PARTIAL BACKLOGGING UNDER INFLATION

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ABSTRACT: In today's time of higher rivalry in the business, there are many conditions, for example, offered concessions in mass acquiring, regularity, higher ordering cost, and so on., which compel a retailer to buy a greater number of amounts than required or surpass the capacity limit. So in this circumstance the retailer needs to buy an additional distribution center named as a leased storehouse to stock the additional amount. In this paper, we have considered a two storehouse (owned and leased storehouse) economic order quantity model for deteriorating items with a selling price dependent demand under the effect of inflation and shortages. Since holding cost of inventory items in leased storehouse is greater than that of owned storehouse, leased storehouses inventory level is depleted due to demand and deterioration. At this time owned storehouse is depleted due to deterioration only. After the inventory level of leased storehouse becomes zero, the inventory level of owned storehouse is depleted due to both demand and deterioration. The shortages are permitted in owned storehouse only which is partially backlogged. This paper aids the retailer in minimizing the total average inventory cost by finding the optimal interval and the optimal order quantity. Finally, a numerical example with sensitivity analysis is given to illustrate the proposed model.

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Key Words: owned storehouse, leased storehouse, partial backlogging, preservation technology, inflation

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1. INTRODUCTION

Hwang and Shinn (1997), Chang et al. (2003), Ouyang et al. (2006), Huang (2006, 2007a, 2007b), Liao (2008), Sana and Chaudhari (2008), Huang and Hsu (2008), Ho et al. (2008), Jaggi and Khanna (2010), Jaggi and Kausar (2011), Jaggi and Mittal (2012) have developed a single warehouse inventory model under the assumption that the accessible warehouse has limitless capacity. The assumption is not realistic. Usually, it is seen that the enterprises or retailers are compelled to purchase more than their stockpiling limits because of offered concession in mass acquiring, to maintain a strategic distance from the ordering cost, and so on. These commodities cannot be stocked in existing storehouse. In these circumstances the business ventures need to buy a leased storehouse to stock the additional requested amount. Typically per unit holding and deteriorating costs in leased storehouse are more noteworthy than the cost in owned storehouse attributable to the extra cost of shielding and holding material so the items ought to be stored first in owned storehouse, also, just surplus stock ought to be stocked in leased storehouse. Thus to decrease the aggregate total inventory cost, it is important to consume the load of leased storehouse first and afterward to expend the stock in owned storehouse. Likewise the expanded limit of owned storehouse abatements the aggregate system cost. So while building up the inventory models the investigation of a two storehouse framework can't be ignored.

Two storehouses inventory model were initially tended to by Hartely (1976) under the supposition that leased storehouse causes a higher stock holding cost than owned storehouse. Thus, items stored in leased storehouse are moved to owned storehouse until stock level in leased storehouse gets to be distinctly zero and after that items stored in owned storehouse are devoured. Goswami and Chaudhuri (1992) broadened this model for shortages and time dependent demand rate. They additionally connected a transportation cost to move the stock from leased storehouse to owned storehouse. However, this model was examined for non-deteriorating items only. Pakkala and Achary (1992) introduced a two warehouses inventory model for deteriorating items. They considered backlogging shortages and finite production for the inventory system. Yang (2004) presented a two warehouse inventory model for deteriorating items with permissible shortages and considered the rate of inflation.

For deteriorating items, a two warehouse inventory model allowing shortages under inflation was studied by Singh et al. (2009). Jaggi and Verma (2010) investigated a two warehouse inventory model by considering inflationary environment and linear trend in demand. Shortages were passable and accumulated totally in this model. A two warehouse inventory model for time varying deterioration and quadratic demand function under finite replenishment rate was determined by Sett et al. (2012). Singh et al. (2013) presented an inventory system of blemished quality items with inflation under two restricted stockpiling limit. Agrawal et al. (2013) recommended a two warehouse inventory model with ramp-type demand for deteriorating items. To build up this model, zero lead-time is assumed and shortages are allowed and partially backlogged at an unvarying rate. Bhunia et al. (2014) investigated a solitary item, two warehouse deteriorating inventory model with particular safeguarding facilities by considering incompletely accumulated shortages over infinite planning horizon. For the plan of the model the rate of demand is assumed as constant and well-known and lead time is also considered as fixed.

For two warehouse inventory system, Jaggi et al. (2015) concentrated the impact of deterioration with blemished quality. They expressed retailer was required to procure leased warehouse to diminish the sufferers brought about by deterioration with enhanced safeguarding facilities, due to not having great facilities in owned warehouse. Jaggi et al. (2016) presented a two warehouse inventory model for non-instantaneous deteriorating items allowing partially backlogged shortages under inflationary condition and they determined the optimal polices by minimizing the present worth of total cost. Palanivel et al. (2016) planned a two warehouse inventory model with noninstantaneously deteriorating items. Authors assumed that demand rate is stock-dependent and Shortages are allowable and partially backlogged in their model. But the attentiveness on price sensitive demand is not stated in their model. An EOQ model for deteriorating items with selling price dependent rate is developed by Rastogi et al. (2017a) to work out the quantity and time of order which can optimize the average total cost of the model. This is an EOQ model for deteriorating items with two warehouse and permissible shortages and occurring shortages are partially backlogged.

Amid the most recent couple of decades, many stock experts extensively have concentrated various aspects of stock displaying by accepting demand rate as fixed. However in authenticity, demand of an item has been for any time in a dynamic state. This gets the consideration of analysts to feel with respect to the fluctuation of demand rate. In the today's focused market, the offering cost of a product is one of the fundamental figures picking the item. The selling price issue accounts for the fact that rise in the selling price of the goods disheartens a replicate demand. Different demand designs have been utilized as a part of the stock demonstrating, for example, consistent, time dependent, stock dependent and selling price dependent. Regularly it is seen that the selling price of the items is most influencing element of demand. For representation, firms may energetically direct their costs to upgrade demand and increment wages. Therefore, the items demand has to depend on the selling price, which makes the review more practical.

Around there, Wee (1997) exhibited a replenishment policy for items with a price dependent demand and a varying deterioration rate. Mondal et al. (2003) recommended an inventory system of deteriorating items wherein price dependent demand rate was assumed. Maiti et al. (2009) gave an inventory system for price dependent demand in stochastic environment. Singh et al. (2011) presented an inventory model based on soft computing with deteriorating items and price dependent demand rate. Jaggi et al. (2014) presented a two warehouse environment inventory model with credit financing for deteriorating items and price-sensitive demand. In this model, shortages were fulfilled completely. Tayal et al. (2015) presented an inventory model for deteriorating items with occasional items and a choice of an option market. In this model, the required items were taken as a function of price and season. Sharma et al. (2015) introduced inventory model for deteriorating items with price-sensitive demand and shortages. Sharma and Chaudhary (2016) developed an inventory model for two warehouses under inflation and shortages in which deterioration rate follows two parameter Weibull distributions and demand rate is price dependent. Rastogi et al. (2017b) discussed an inventory model for deteriorating items with price sensitive demand, possible cases of permissible delay and cash discount under credit limit policy.

In several developed models, the consideration is not given to the shortages when stock out and if the specialists considered shortages they expected it totally accumulated or totally lost. Both of these conditions don't fulfill the state of backlogging totally. Since a few clients return to finish their demand happening during stock out and some other restless clients make their buys from whatever other spots. Dave (1989) proposed a lot sizing inventory model with permissible shortages and linear trend demand. Ouyang et al. (1999) explored reductions policies for lead time and ordering cost in continuous assessment inventory systems with partially backlogged shortages. An inventory system for deterioration of items with exponentially declining demand and partially backlogged shortage was developed by Ouyang et al. (2005). Chung and Huang (2007) and Liang and Zhou (2011) developed two warehouse inventory models for deteriorating items. In both the works, the demand is considered as constant and shortages are not allowed. In business world, Stock out situation plays an important role. Due to some unavailable circumstances, stock out situation may occur in any business. Chern et al. (2008) presented a partially backlogged lot sizing model for deteriorating goods with variable demand. Skouri et al. (2009) proposed an inventory model by in view of ramp-type demand, shortages and Weibull deterioration. Valliathal and Uthayakumar (2011) investigated the effects of inflation and time discounting on an EOQ model for perishable items with both stock varying and time varying demand. In their works, shortages are allowed and backlogged partially. Taleizadeh and Pentico (2013) introduced an EOQ model with a well-known price increase and partial backlogging. A two echelon supply chain model for deteriorating items with efficient investment in preservation technology was developed by Tayal et al. (2014). In their model, the shortages are allowed and partially backlogged. Shastri et al. (2015) explored an inventory model for deteriorating items in view of trade credit policy and ramp-type demand. They allowed shortages for this model and unfulfilled demand was backlogged partially. They also assumed that the deterioration rate is taken as linear increasing function of time. San-Jos et al. (2015) examined an EOQ inventory model with partial backlogging. At the time of stock out period, shortages are allowable and a part of demand is backordered. Singh et al. (2016) analyzed an inventory policy for deteriorating items wherein demand is stock dependent and the retailer invests in preservation technology to reduce the rate of items deterioration. This was developed under the realistic conditions of demand, permissible credit period, partial backlogging and variable ordering cost. Recently, Khanna et al. (2016) analyzed an inventory model for imperfect quality and deteriorating items allowing for permissible delay in payments and permissible shortages and happening shortages are considered as completely backlogged. Saha and Sen (2017) studied the effect on optimality when deterioration is considered as three different probability functions. Partial backlogging shortage and negative exponential demand were introduced in this model.

Kumar and Rajput (2016) developed partially backlogging inventory model for deteriorating items with probabilistic deterioration rate and ramp type demand under stock dependent consumption rate. Shortages are backlogged completely and the backlogging rate of unfulfilled demand is assumed as a function of waiting time. Also, the effect of inflation and time value of money is taken into account. The ramp type demand is a demand which increases up to a certain time and after that it becomes stable or constant. An economic order quantity model for deteriorating products was developed by Khurana and Chaudhary (2016) to find out optimal selling price and optimal ordering quantity. In this study, two different and possible cases of partial backlogging are studied. In the first case, the rate of backlogging is constant and in the second case, the rate of backlogging is assumed to be dependent on the waiting time up to the arrival of next lot. Pandey et al. (2017) developed an inventory model for deteriorating items with quantity discount, selling price dependent demand and partial backlogging to determine the optimal ordering quantity for retailers.

From above writing it is watched that less interest has been paid by the researchers in developing two-warehouse inventory model with price-sensitive demand. Thus, in this present model we join all specified elements with the selling price dependent demand. This is an EOQ model for two deteriorating warehouse with admissible shortages and happening shortages are backlogged partially under inflation and preservation technology. The remaining sections of the paper are organized as follows. Section 2 describes the assumptions and notations used in this model. Section 3 presents the mathematical formulation of the inventory model. In section 4, the convexity of the total average cost function is derived. Section 5 proposes an algorithm to find the optimal solutions. In section 6, a numerical example with sensitivity analysis is given. Finally, conclusions are made and future research guidelines are outlined in section 7.

2. ASSUMPTIONS AND NOTATIONS

2.1. ASSUMPTIONS

The following assumptions are considered to develop the present model.

1. The replenishment rate is considered as infinite.
2. The owned storehouse has a limited capacity of W units.
3. The rented warehouse has unlimited capacity.
4. The lead time is assumed to be zero.
5. The demand rate is a function of selling price.
6. The items considered in this model are deteriorating in nature.
7. The items are stored in leased storehouse only after filling owned storehouse.
8. The items kept in leased storehouse will be consumed first.
9. The shortages are allowed and partially backlogged.
10. Holding cost per unit in leased storehouse is greater in comparison of holding cost per unit in owned storehouse.

2.2. NOTATIONS

The following notations are considered through the present model.

$I_{lw}(t)$	Inventory level at time t in leased storehouse.
$I_{ow}(t)$	Inventory level at time t in owned storehouse.
θ	Actual rate of deterioration
ϵ	Cost of preservation technology for reducing rate of deterioration in order to preserve the items,ϵ
λ	Resultant deterioration rate, where reduced deterioration rate, a function of ϵ
a	Initial demand rate
b	Positive demand parameter
R	Inflationary rate (the difference between capital cost and cost after inflation
t_1	The time at which inventory level in leased storehouse becomes zero
t_2	Time at which inventory level becomes zero in owned storehouse.
s	Selling price per unit
Q_1	Initial stock level
Q_2	Backordered quantity during stock out
T	Cycle time
p_c	Purchasing cost per unit
β	Rate of backlogging
h_{lw}	Holding cost per unit in leased storehouse.
h_{ow}	Holding cost per unit in owned storehouse.
D_c	Per unit deterioration cost
s_c	Per unit shortage cost
O_l	Per unit opportunity cost due to lost sale
TAC	Total average cost

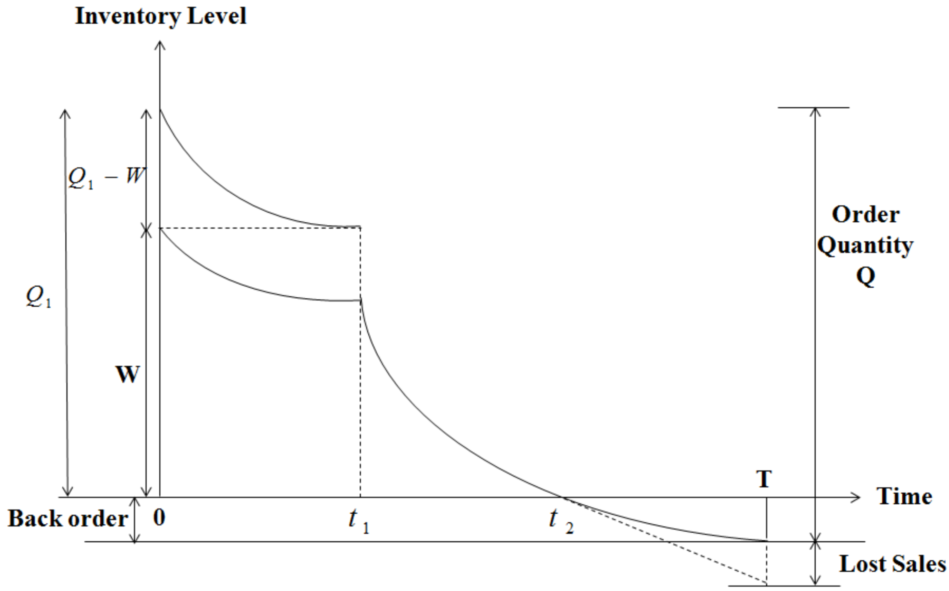

Figure 1: Inventory time graph of two storehouse inventory model

3. MATHEMATICAL MODELING OF THE INVENTORY MODEL

In the beginning Q units are received in stock, out of which Q_2 units are utilized to satisfy backlogged demand and Q_1 units are the initial stock level. Since capacity of owned storehouse is only W units and $Q_1 >$ W, so remaining (Q_1 - W) units are stored in a rented warehouse. Now since holding cost in leased storehouse is greater compared with holding cost in owned storehouse, the items in leased storehouse will be consumed first. In this duration inventory level in owned storehouse is reduced because of deterioration only. At t $= t_1$ inventory level in leased storehouse becomes zero after satisfying the demand and deterioration. During $[t_1 \ ,t_2]$ stock is available only in owned storehouse. At t $= t_2$ inventory level in owned storehouse also becomes zero and after that shortage occurs and is shown in Figure 1.

The differential equations governing the inventory level at any time $'t'$ during the cycle $(0, T)$ are given below:

$$\frac{dI_{lw}(t)}{dt} + \lambda I_{lw}\,(t) = -(a - bs), \quad 0 \leq t \leq t_1; \tag{1}$$

with boundary condition $I_{lw}(t_1) = 0.$

$$\frac{dI_{ow}(t)}{dt} + \lambda I_{ow}(t) = 0, \quad 0 \le t \le t_1; \tag{2}$$

with boundary condition $I_{ow}(0) = W$

$$\frac{dI_{ow}(t)}{dt} + \lambda I_{ow}(t) = -(a - bs), \quad t_1 \le t \le t_2; \tag{3}$$

with boundary condition $I_{ow}(t_2) = 0.$

$$\frac{dI_s(t)}{dt} = -\beta(a - bs), \quad t_2 \le t \le T \tag{4}$$

with boundary condition $I_s(t_2) = 0.$

Solutions of the differential equations (1), (2), (3) and (4) are given below:

$$I_{lw}(t) = \frac{(a - bs)}{\lambda}\left[e^{\lambda(t_1 - t)} - 1\right], \quad 0 \le t \le t_1; \tag{5}$$

$$I_{ow}(t) = We^{-\lambda t}, \quad 0 \le t \le t_1; \tag{6}$$

$$I_{ow}(t) = \frac{(a - bs)}{\lambda}\left[e^{\lambda(t_2 - t)} - 1\right], \quad t_1 \le t \le t_2; \tag{7}$$

$$I_s(t) = \beta(a - bs)(t_2 - t), \quad t_2 \le t \le T. \tag{8}$$

At initial stage an order of $Q_1 + Q_2$ units is made out of which the Q_2 units are used to meet the backordered quantity and the remaining Q_1units are stored as the initial stock level for next cycle.

Since the owned warehouse has a limited capacity of W units, so if the stock level $Q_1 = W$, then the remaining quantity $Q_1 - W$ will be stored in rented warehouse.

Since $I_{lw}(0) = Q_1 - W$, from equation (5) we can obtain the initial stock quantity Q_1:

$$Q_1 = W + \frac{(a - bs)}{\lambda}\left[e^{\lambda t_1} - 1\right] \tag{9}$$

When $t = t_1$, the equations (6) and (7) are equal and hence we obtain the capacity of owned warehouse:

$$W = \frac{(a - bs)}{\lambda}\left[e^{\lambda t_2} - e^{\lambda t_1}\right] \tag{10}$$

Using Taylor approximation ($e^x = 1 + x$) in equation (10), one can get

$$t_1 = t_2 - \frac{W}{(a-bs)} \tag{11}$$

The backordered quantity Q_2 can be calculated from equation (8) when $t = T$.

$$Q_2 = -I_s\,(T) = \beta(a-bs)(T-t_2) \tag{12}$$

Thus the order size during the total time interval $(0,\ T)$ is given by

$$Q = Q_1 + Q_2 = W + \frac{(a-bs)}{\lambda}\left[e^{\lambda t_1} - 1\right] + \ \beta(a-bs)(T-t_2) \tag{13}$$

DIFFERENT COST ANALYSIS

(i) Present value of purchase cost (PC):

$$\begin{aligned} PC &= p_c \int_0^T Qe^{-Rt}dt \\ &= \frac{p_c}{R}\left[W + \frac{(a-bs)}{\lambda}\left(e^{\lambda t_1} - 1\right) + \ \beta(a-bs)(T-t_2)\right]\left[1 - e^{-RT}\right]. \end{aligned} \tag{14}$$

(ii) Present value of holding cost (HC):

Case (i): when stock is kept at leased warehouse, the inventory holding cost is calculated by:

$$\begin{aligned} HC_{LW} &= h_{lw} \int_0^{t_1} I_{lw}\,(t)\,e^{-Rt}dt \\ &= h_{lw}\frac{(a-bs)}{\lambda R(\lambda+R)}\left\{\lambda\left(e^{-Rt_1} - 1\right) + R\left(e^{\theta t_1} - 1\right)\right\}. \end{aligned} \tag{15}$$

Case (ii): when stock is kept at owned warehouse, the inventory holding cost is calculated by:

$$\begin{aligned} HC_{OW} &= h_{ow}\left\{\int_0^{t_1} I_{ow}\,(t)\,e^{-Rt}dt + \int_{t_1}^{t_2} I_{ow}\,(t)\,e^{-Rt}dt\right\} \\ &= \frac{h_{ow}}{(\lambda+R)}\left\{W\left(1 - e^{-(\lambda+R)t_1}\right)\right. \\ &\quad \left. + \frac{(a-bs)}{\lambda R}\left[\lambda\left(e^{-Rt_2} - e^{-Rt_1}\right) + Re^{-Rt_1}\left(e^{\lambda(t_2-t_1)} - 1\right)\right]\right\}. \end{aligned} \tag{16}$$

(iii) Present value of deterioration cost (DC):

Case (i): when the stock of items is stored in leased warehouse, the inventory deterioration cost is given by:

$$DC_{LW} = D_c \int_0^{t_1} \lambda I_{lw}(t)\, e^{-Rt} dt$$
$$= \frac{D_c(a-bs)}{R(\lambda+R)} \left\{ \lambda \left(e^{-Rt_1} - 1 \right) + R \left(e^{\lambda t_1} - 1 \right) \right\}. \quad (17)$$

Case (ii): when the stock of items is stored in owned warehouse, the inventory deterioration cost is given by:

$$\begin{aligned} DC_{OW} =& D_c \int_0^{t_1} \lambda I_{ow}(t)\, e^{-Rt} dt \\ &+ D_c \int_{t_1}^{t_2} \lambda I_{ow}(t)\, e^{-Rt} dt \\ =& \frac{D_c W \lambda}{(\lambda+R)} \left[1 - e^{-(\lambda+R)t_1} \right] \\ &+ \frac{D_c(a-bs)}{R(\lambda+R)} \left\{ \lambda \left(e^{-Rt_2} - e^{-Rt_1} \right) \right. \\ &\left. - Re^{-Rt_1} \left(1 + e^{\lambda(t_2 - t_1)} \right) \right\}. \end{aligned} \quad (18)$$

(iv) Present value of shortage cost (SC):

$$SC = -s_c \int_{t_2}^{T} I_s(t)\, e^{-Rt} dt$$
$$= \frac{\beta s_c(a-bs)}{R^2} \left\{ e^{-RT} \left(1 - Rt_2 + RT \right) - e^{-Rt_2} \right\}. \quad (19)$$

(v) Present value of opportunity cost due to lost sales (OL):

$$OL = o_l \int_{t_2}^{T} (1-\beta)(a-bs)\, e^{-Rt} dt$$
$$= \frac{o_l (1-\beta)(a-bs)}{R} \left[e^{-Rt_2} - e^{-RT} \right]. \quad (20)$$

(vi) Present value of ordering cost (OC):

$$OC = o_c \int_0^{T} e^{-Rt} dt = \frac{o_c}{R} \left(1 - e^{-RT} \right). \quad (21)$$

Therefore, the average total cost for the present model during a given cycle is:

$$TAC = \frac{1}{T}\left[PC + HC_{LW} + HC_{OW} + DC_{LW} + DC_{OW} + SC + OL + OC\right],$$

$$\begin{aligned} TAC =& \frac{p_c}{R}\left[W + \frac{(a-bs)}{\lambda}\left(e^{\lambda t_1} - 1\right) + \ \beta(a-bs)(T-t_2)\right]\left[1 - e^{-RT}\right] \\ &+ h_{lw}\frac{(a-bs)}{\lambda R(\lambda+R)}\left\{\lambda\left(e^{-Rt_1} - 1\right) + R\left(e^{\theta t_1} - 1\right)\right\} \\ &+ \frac{h_{ow}}{(\lambda+R)}\left\{W\left(1 - e^{-(\lambda+R)t_1}\right) + \frac{(a-bs)}{\lambda R}\right. \\ &\times\left.\left[\lambda\left(e^{-Rt_2} - e^{-Rt_1}\right) + Re^{-Rt_1}\left(e^{\lambda(t_2-t_1)} - 1\right)\right]\right\} \\ &+ \frac{D_c(a-bs)}{R(\lambda+R)}\left\{\lambda\left(e^{-Rt_1} - 1\right) + R\left(e^{\lambda t_1} - 1\right)\right\} \\ &+ \frac{D_c W\lambda}{(\lambda+R)}\left[1 - e^{-(\lambda+R)t_1}\right] + \frac{D_c(a-bs)}{R(\lambda+R)} \\ &\times\left\{\lambda\left(e^{-Rt_2} - e^{-Rt_1}\right) - Re^{-Rt_1}\left(1 + e^{\lambda(t_2-\ t_1)}\right)\right\} \\ &+ \frac{\beta s_c(a-bs)}{R^2}\left\{e^{-RT}\left(1 - Rt_2 + RT\right) - e^{-Rt_2}\right\} \\ &+ \frac{o_l\left(1-\beta\right)\left(a-bs\right)}{R}\left[e^{-Rt_2} - e^{-RT}\right] + \frac{o_c}{R}\left(1 - e^{-RT}\right). \end{aligned} \tag{22}$$

With the help of equation (11), average total cost (TAC) can be written in terms of t_2 and T which is given by

$$\begin{aligned} TAC(t_2,\ T) =& \frac{p_c}{R}\left[W + \frac{(a-bs)}{\lambda}\left(e^{\lambda(t_2-\xi)} - 1\right) + \ \beta(a-bs)(T-t_2)\right]\left[1 - e^{-RT}\right] \\ &+ h_{lw}\frac{(a-bs)}{\lambda R(\lambda+R)}\left\{\lambda\left(e^{-R(t_2-\xi)} - 1\right) + R\left(e^{\theta(t_2-\xi)} - 1\right)\right\} \\ &+ \frac{h_{ow}}{(\lambda+R)}\left\{W\left(1 - e^{-(\lambda+R)(t_2-\xi)}\right)\right. \\ &+\left.\frac{(a-bs)}{\lambda R}\left[\lambda\left(e^{-Rt_2} - e^{-R(t_2-\xi)}\right) + Re^{-R(t_2-\xi)}\left(e^{\lambda\xi} - 1\right)\right]\right\} \\ &+ \frac{D_c(a-bs)}{R(\lambda+R)}\left\{\lambda\left(e^{-R(t_2-\xi)} - 1\right) + R\left(e^{\lambda(t_2-\xi)} - 1\right)\right\} \\ &+ \frac{D_c W\lambda}{(\lambda+R)}\left[1 - e^{-(\lambda+R)(t_2-\xi)}\right] \\ &+ \frac{D_c(a-bs)}{R(\lambda+R)}\left\{\lambda\left(e^{-Rt_2} - e^{-R(t_2-\xi)}\right) - Re^{-R(t_2-\xi)}\left(1 + e^{\lambda\xi}\right)\right\} \end{aligned}$$

$$+ \frac{\beta s_c(a-bs)}{R^2}\left\{e^{-RT}\left(1-Rt_2+RT\right)-e^{-Rt_2}\right\}$$
$$+ \frac{o_l\,(1-\beta)\,(a-bs)}{R}\left[e^{-Rt_2}-e^{-RT}\right]+\frac{o_c}{R}\left(1-e^{-RT}\right), \quad (23)$$

where $\xi = \frac{W}{(a-bs)}$.

To make the solution method simple, the average total cost (TAC), equation (23), can be simplified after algebraic simplification as:

$$\begin{aligned}
TAC(t_2,\ T) = &\left[\frac{p_cW+o_c-D_c(a-bs)}{R}-\frac{(a-bs)\,(p_c+h_{rw})}{\lambda R}\right.\\
&\left.+\frac{W(h_{ow}+\lambda D_c)}{(\lambda+R)}\right]\frac{1}{T}\\
&-\left[\frac{p_c\beta(a-bs)}{R}\right]\frac{t_2}{T}+\left[\frac{p_c}{\lambda R}+\frac{h_{rw}}{\lambda\,(\lambda+R)}+\frac{D_c}{(\lambda+R)}\right]\\
&\times(a-bs)e^{-\lambda\xi}\left(\frac{e^{\theta t_2}}{T}\right)\\
&+\left[\frac{p_c(a-bs)}{\lambda R}+\frac{\beta s_c(a-bs)}{R^2}-\frac{p_cW+o_l\,(a-bs)\,(1-\beta)+o_c}{R}\right]\\
&\times\frac{e^{-RT}}{T}\\
&+\left[\frac{\beta\,(a-bs)\,(p_c-s_c)}{R}\right]\left(\frac{e^{-RT}(t_2-T)}{T}\right)-\left[\frac{p_c(a-bs)e^{-\lambda\xi}}{\lambda R}\right]\\
&\times\frac{e^{(\theta t_2-RT)}}{T}\\
&+\left[\frac{(a-bs)\,e^{R\xi}(h_{rw}+\lambda D_c)}{R(\lambda+R)}\right.\\
&+\frac{(a-bs)\,(h_{ow}+\lambda D_c)\left[\lambda-e^{R\xi}\left(\lambda+R-Re^{\lambda\xi}\right)\right]}{\lambda R(\lambda+R)}\\
&\left.-\frac{\beta s_c(a-bs)}{R^2}+\frac{o_l\,(a-bs)\,(1-\beta)}{R}\right]\frac{e^{-Rt_2}}{T}\\
&-\left[\frac{We^{(\lambda+R)\xi}\,(h_{ow}+\lambda D_c)}{(\lambda+R)}\right]\frac{e^{-(\theta+R)t_2}}{T}+\frac{p_c\beta(a-bs)}{R}.
\end{aligned} \quad (24)$$

4. CONVEXITY OF THE TOTAL AVERAGE COST FUNCTION

In this section, we discuss the convexity of the total average cost function. Here TAC is a function of two variables 't_2' and 'T'. So to compute the minimum value of TAC we have to compute optimal value of 't_2' and 'T'.

Solving

$$\frac{dTAC(t_2,\ T)}{dt_2} = 0 \text{ and } \frac{dTAC(t_2,\ T)}{dT} = 0$$

simultaneously, we get optimum value of 't_2' and 'T'.

The necessary and sufficient condition for minimizing the total average cost $TAC(t_2,\ T)$ is the Hessian matrix

$$H = \begin{bmatrix} \dfrac{d^2(TAC)}{dt_2^2} & \dfrac{d^2(TAC)}{dtdT} \\ \dfrac{d^2(TAC)}{dTdt} & \dfrac{d^2(TAC)}{dT^2} \end{bmatrix}$$

is a positive definite.

Differentiating the equation (24) with respect to t_2, we receive

$$\begin{aligned} \frac{dTAC(t_2,\ T)}{dt_2} =& \frac{1}{T}\Bigg[- \left[\frac{p_c\beta(a-bs)}{R}\right] \\ &+ \left[\frac{p_c}{\lambda R} + \frac{h_{rw}}{\lambda(\lambda+R)} + \frac{D_c}{(\lambda+R)}\right]\lambda(a-bs)e^{-\lambda\xi}e^{\lambda t_2} \\ &- \left[\frac{p_c(a-bs)e^{-\lambda\xi}}{\lambda R}\right]\lambda e^{-RT}e^{\lambda t_2} \\ &+ \left[\frac{\beta(a-bs)(p_c-s_c)}{R}\right]e^{-RT} \\ &- \left[\frac{(a-bs)e^{R\xi}(h_{rw}+\lambda D_c)}{R(\lambda+R)}\right. \\ &+ \frac{(a-bs)(h_{ow}+\lambda D_c)\left[\lambda - e^{R\xi}\left(\lambda+R-Re^{\lambda\xi}\right)\right]}{\lambda R(\lambda+R)} \\ &\left. - \frac{\beta s_c(a-bs)}{R^2} + \frac{o_l(a-bs)(1-\beta)}{R}\right]Re^{-Rt_2} \\ &+ \left[\frac{We^{(\lambda+R)\xi}(h_{ow}+\lambda D_c)}{(\lambda+R)}\right](\lambda+R)e^{-(\lambda+R)t_2}\Bigg]. \end{aligned} \tag{25}$$

Again differentiating with respect to t_2 we have

$$\begin{aligned}\frac{d^2TAC(t_2,\ T)}{dt_2^2} =& \frac{1}{T}\Bigg[\left[\frac{p_c}{\lambda R}+\frac{h_{rw}}{\lambda\left(\lambda+R\right)}+\frac{D_c}{\left(\lambda+R\right)}\right]\lambda^2\left(a-bs\right)e^{-\lambda\xi}e^{\lambda t_2}\\
&-\left[\frac{p_c(a-bs)e^{-\lambda\xi}}{\lambda R}\right]\lambda^2 e^{-RT}e^{\lambda t_2}\\
&+\Bigg[\frac{(a-bs)\,e^{R\xi}(h_{rw}+\lambda D_c)}{R(\lambda+R)}\\
&+\frac{(a-bs)\left(h_{ow}+\lambda D_c\right)\left[\lambda-e^{R\xi}\left(\lambda+R-Re^{\lambda\xi}\right)\right]}{\lambda R(\lambda+R)}\\
&-\frac{\beta s_c(a-bs)}{R^2}\\
&+\frac{o_l\left(a-bs\right)\left(1-\beta\right)}{R}\Bigg]R^2e^{Rt_2}\\
&-\left[\frac{We^{(\lambda+R)\xi}\left(h_{ow}+\lambda D_c\right)}{(\lambda+R)}\right]\left(\lambda+R\right)^2e^{-(\lambda+R)t_2}\Bigg]. \qquad (26)\end{aligned}$$

Differentiating equation (24) with respect to T, we get

$$\begin{aligned}\frac{dTAC(t_2,\ T)}{dT} =& \frac{-1}{T^2}\Bigg[\frac{p_cW+o_c-D_c(a-bs)}{R}-\frac{(a-bs)\left(p_c+h_{rw}\right)}{\lambda R}\\
&+\frac{W(h_{ow}+\lambda D_c)}{(\lambda+R)}-\left[\frac{p_c\beta(a-bs)}{R}\right]t_2\\
&+\left[\frac{p_c}{\lambda R}+\frac{h_{rw}}{\lambda\left(\lambda+R\right)}+\frac{D_c}{\left(\lambda+R\right)}\right](a-bs)e^{-\lambda\xi}e^{\lambda t_2}+\frac{p_c(a-bs)}{\lambda R}\\
&+\left[\frac{\beta s_c(a-bs)}{R^2}-\frac{p_cW+o_l\left(a-bs\right)\left(1-\beta\right)+o_c}{R}\right]\left(1+TR\right)e^{-RT}\\
&-\left[\frac{p_c(a-bs)e^{-\lambda\xi}}{\lambda R}\right]e^{\lambda t_2}e^{-RT}\\
&+\left(\frac{\beta\left(a-bs\right)\left(p_c-s_c\right)}{R}\right)\left(TRt_2-T^2R+t_2\right)\\
&+\Bigg[\frac{(a-bs)\,e^{R\xi}(h_{rw}+\lambda D_c)}{R(\lambda+R)}\\
&+\frac{(a-bs)\left(h_{ow}+\lambda D_c\right)\left[\lambda-e^{R\xi}\left(\lambda+R-Re^{\lambda\xi}\right)\right]}{\lambda R(\lambda+R)}\\
&-\frac{\beta s_c(a-bs)}{R^2}\\
&+\frac{o_l\left(a-bs\right)\left(1-\beta\right)}{R}\Bigg]e^{-Rt_2}\end{aligned}$$

$$-\left[\left[\frac{We^{(\beta+R)\xi}\left(h_{ow}+\lambda D_c\right)}{(\lambda+R)}\right]e^{-(\lambda+R)t_2}\right]. \tag{27}$$

Again differentiating with respect to T, we get

$$\begin{aligned}\frac{d^2TAC(t_2,\ T)}{dT^2} =& \frac{1}{T^3}\Bigg[2\left[\frac{p_cW+o_c-D_c(a-bs)}{R}-\frac{(a-bs)\left(p_c+h_{rw}\right)}{\lambda R}\right.\\
&\left.+\frac{W(h_{ow}+\lambda D_c)}{(\lambda+R)}\right]-2\left[\frac{p_c\beta(a-bs)}{R}\right]t_2\\
&+2\left[\frac{p_c}{\lambda R}+\frac{h_{rw}}{\lambda\left(\lambda+R\right)}+\frac{D_c}{(\lambda+R)}\right](a-bs)\,e^{-\lambda\xi}e^{\lambda t_2}\\
&-2\left[\frac{We^{(\lambda+R)\xi}\left(h_{ow}+\lambda D_c\right)}{(\lambda+R)}\right]e^{-(\lambda+R)t_2}\\
&+\left[\frac{p_c\left(a-bs\right)}{\lambda R}+\frac{\beta s_c\left(a-bs\right)}{R^2}\right.\\
&\left.-\frac{p_cW+o_l\left(a-bs\right)\left(1-\beta\right)+o_c}{R}\right]\left(R^2T^2+2RT+2\right)e^{-RT}\\
&-\left[\frac{p_c(a-bs)e^{-\lambda\xi}}{\lambda R}\right]\left(R^2T^2+2RT+2\right)e^{-RT}e^{\lambda t_2}\\
&+\left[\frac{\beta\left(a-bs\right)\left(p_c-s_c\right)}{R}\right]\\
&\left[-R^2T^3+\left(Rt_2-1\right)RT^2+\left(2t_2+1\right)RT\right]e^{-RT}\\
&+2\left[\frac{(a-bs)\,e^{R\xi}(h_{rw}+\lambda D_c)}{R(\lambda+R)}\right.\\
&+\frac{(a-bs)\left(h_{ow}+\lambda D_c\right)\left[\lambda-e^{R\xi}\left(\lambda+R-Re^{\lambda\xi}\right)\right]}{\lambda R(\lambda+R)}\\
&\left.-\frac{\beta s_c(a-bs)}{R^2}+\frac{o_l\left(a-bs\right)\left(1-\beta\right)}{R}\right]e^{-Rt_2}\Bigg].\end{aligned} \tag{28}$$

Differentiating equation (27) with respect to t_2 we get

$$\begin{aligned}\frac{d^2TAC(t_2,\ T)}{dt_2} =& \frac{-1}{T^2}-\left[\frac{p_c\beta(a-bs)}{R}\right]-\left[\frac{p_c}{\lambda R}+\frac{h_{rw}}{\lambda\left(\lambda+R\right)}+\frac{D_c}{(\lambda+R)}\right]\\
&\times\lambda\left(a-bs\right)e^{-\lambda\xi}e^{\lambda t_2}\\
&+\left[\frac{p_c(a-bs)e^{-\lambda\xi}}{\lambda R}\right]\lambda e^{\lambda t_2-RT}\left(1+TR\right)\\
&-\left[\frac{\beta\left(a-bs\right)\left(p_c-s_c\right)}{R}\right]\left(1+TR\right)\end{aligned}$$

$$
\begin{aligned}
&+\left[\frac{(a-bs)\,e^{R\xi}(h_{rw}+\lambda D_c)}{R(\lambda+R)}\right.\\
&+\frac{(a-bs)\,(h_{ow}+\lambda D_c)\left[\lambda-e^{R\xi}\left(\lambda+R-Re^{\lambda\xi}\right)\right]}{\lambda R(\lambda+R)}\\
&-\frac{\beta s_c(a-bs)}{R^2}\\
&+\left.\frac{o_l\,(a-bs)\,(1-\beta)}{R}\right]Re^{-Rt_2}\\
&-\left[\frac{We^{(\lambda+R)\xi}\,(h_{ow}+\lambda D_c)}{(\lambda+R)}\right](\lambda+R)e^{-(\lambda+R)t_2}.
\end{aligned}
\tag{29}
$$

Since

$$
D\,(H_1)=\frac{\partial^2(TAC)}{\partial t_2^2}>0
$$

and

$$
D\,(H_2)=\left(\frac{\partial^2(TAC)}{\partial t_2^2}\right)\left(\frac{\partial^2(TAC)}{\partial T^2}\right)-\left(\frac{\partial^2(TAC)}{\partial T\partial t}\right)^2>0,
$$

then H is positive definite.

5. THE ALGORITHM

Step 1: Start.
Step 2: Assign a value for each parameters θ, ε, λ, a, b, s, p_c, β, h_{lw}, h_{ow}, D_c, s_c, o_l, R and W.
Step 3: Solve the equations $\frac{\partial TAC(t_2,\ T)}{\partial t_2}=0$ and $\frac{\partial TAC(t_2,\ T)}{\partial T}=0$ simultaneously and obtain the values of t_2 and T using step-2.
Step 4: Evaluate D(H_1), D(H_2) and $TAC(t_2,\ T)$.
Step 5: If the values of D(H_1) and D(H_2) are greater than zero, then the corresponding total average cost $TAC(t_2,\ T)$ is minimum (denoted by $TAC^*(t_2,\ T)$) and corresponding values of t_2 and T are called optimal values which are denoted by t_2^* and T^* respectively. Otherwise go to step-2 and choose another set of values of the parameters.
Step 6: Repeat step-3 to step-5 until we get $TAC^*(t_2,\ T)$, t_2^* and T^*.
Step 7: End.

6. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

6.1. NUMERICAL EXAMPLE

A numerical example is established with the help of following input parameters: $\theta = 0.05$, $\varepsilon = 55$, $\lambda = 0.03$, $a = 60$units, $b = 0.7$, $s = \$\ 25$/unit, $p_c = \$\ 5$/unit, $\beta = 0.7$,$h_{lw} = \$0.05$/unit, $h_{ow} = \$0.04$/unit, $R = 0.06$, $D_c = \$\ 14$/unit, $s_c = \$\ 12$/unit, $o_l = \$\ 15$/unit and $W = 150$units. The output of the model by using maple mathematical software is given below: The optimum time at which the inventory level reaches zero is obtained by:

$$t_2^* = 1.137$$

The optimum cycle time is obtained by:

$$T^* = 1.752$$

The optimum value of the total cost is obtained by:

$$TAC^*\left(t_2,\ T\right) = \$\ 2,357.52$$

The optimum economic order quantity is obtained by:

$$Q^* \approx 153.$$

The amount of backordered quantity is

$$Q_2 \approx 11.$$

6.2. SENSITIVITY ANALYSIS

The change in the values of parameters occurs due to uncertainties in any decision-making circumstances. In order to inspect the inference of these changes, the sensitivity analysis will be of huge help in decision-making. In this section, the sensitivity analysis of various parameters has been analyzed. We change one parameter at a time keeping the other parameters unchanged. The results of sensitivity analysis are summarized in Table 1.

Based on our numerical results, we obtain the following managerial phenomena:

Table 1: Sensitivity analysis for various inventory parameters

parameters	**Parameters value**	t_2^*	Q^*	TAC^*
s	25	1.137	153	2357.52
	30	1.726	142	3841.12
	35	2.001	131	4911.71
p_c	5	1.137	153	2357.52
	8	0.913	196	1983.32
	11	1.529	230	1321.94
h_{lw}	0.05	1.137	153	2357.52
	0.07	1.256	201	2161.37
	0.09	1.438	292	1889.56
h_{ow}	0.04	1.137	153	2357.52
	0.06	1.876	219	1525.81
	0.08	2.083	326	971.85
D_c	14	1.137	153	2357.52
	18	1.175	168	2221.67
	22	1.193	181	2073.21
s_c	12	1.137	153	2357.52
	22	1.083	134	2825.73
	32	0.946	107	3361.28

OBSERVATIONS

1. When the selling price (s) is increasing, the optimal time at which the inventory level becomes vanish in owned storehouse (t_2^*) and the optimal total average cost (TAC^*) are also increasing, on the other hand, the optimal order quantity (Q^*) is decreasing. The selling price is highly sensitive with the total average cost.

2. When the purchasing price (p_c) is increasing, the optimal time at which the inventory level becomes vanish in owned storehouse (t_2^*) and the optimal order quantity (Q^*) are also increasing and the optimal total average cost (TAC^*) is decreasing. Since holding cost of inventory items

in leased storehouse is greater than owned storehouse, the time (t_2^*) is to be reduced.

3. When the holding cost of leased storehouse (h_{lw}) is increasing, the optimal time at which the inventory level becomes vanish in owned storehouse (t_2^*) and the optimal order quantity (Q^*) are also increasing and the optimal total average cost (TAC^*) is decreasing.

4. When the holding cost of owned storehouse (h_{ow}) is increasing, the optimal time at which the inventory level becomes vanish in owned storehouse (t_2^*) and the optimal order quantity (Q^*) are also increasing and the optimal total average cost (TAC^*) is decreasing.

5. When the cost of deterioration (D_c) is increasing, the optimal time at which the inventory level becomes vanish in owned storehouse (t_2^*), the optimal order quantity (Q^*) and the optimal total average cost (TAC^*) is also increasing. It also shows that when deterioration increases one should order less whereas if deterioration decreases, one should order more. Hence the present model is useful to reduce the preset worth of total average cost.

6. If shortage cost (s_c) is increasing, then the optimal time at which the inventory level becomes vanish in owned storehouse (t_2^*) and the optimal order quantity (Q^*) are decreasing and the optimal total average cost (TAC^*) is increasing.

7. CONCLUSION

We have established a two-storehouse inventory model for deteriorating items with preservation technology and selling price dependent demand under the effect of inflation. Also in this model, shortages are permitted and partially backlogged. Since the capacity of owned storehouse is limited, vendor has to stock the extra quantity in any leased storehouse practice. The leased storehouse is assumed to offer better preserving facilities than the owned storehouse resulting in a lower rate of deterioration and is assumed to charge higher holding cost than the owned storehouses. The problem of inventory systems under inflationary conditions has received attention in recent years. Due to high inflation and consequent sharp decline in the purchasing power of money, especially in the developing countries, the financial situation has been changed and so it is impossible to ignore the effect of inflation. This model aids to develop an inventory model to find out the optimum quantity and the time of order which can optimize the total average cost of the system. Furthermore, a numerical example is provided to illustrate the proposed model, sensitivity analysis is carried out with respect to the key parameters and useful managerial insights are obtained. The proposed model incorporates some realistic features that are likely to be associated with some kinds of inventory. Furthermore, this model can be adopted in the inventory control of retail business such as food industries, domestic goods, automobile, electronic components, etc. This model can be formulated in several ways. For example, we may extend the model by considering permissible delay in payments, two level trade credit policy, quantity discount, holding cost is fluctuating with time, multi-item, etc.

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