PULSATILE FLOW OF NON-NEWTONIAN NANOFLUID IN A POROUS SPACE WITH THERMAL RADIATION: AN APPLICATION TO THE BLOOD FLOW

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ABSTRACT: In the present analysis, the pulsatile flow of blood carrying gold nanoparticles of Eyring-Powell nanofluid in a porous space with thermal radiation is investigated. We considered the base fluid as blood which is non-Newtonian and gold as nanoparticle. The governing flow equations are solved analytically. The effects of various emerging parameters on velocity, temperature and heat transfer rate of nanofluid are discussed graphically. It is found that the velocity of nanofluid decreases for a given increase in non-Newtonian parameter and nanoparticle volume fraction. Further, the heat transfer rate increases with increasing nanoparticles volume fraction at the lower wall.

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Key Words: pulsatile flow, gold nanoparticles, Eyring-Powell nanofluid, thermal radiation, Darcy number, porous space

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1. INTRODUCTION

The investigations of the pulsatile flows in a channel or pipe have gained much attention due to its importance in biological and industrial applications such as circulatory system, respiratory system, vascular diseases, reciprocating pumps, IC engines and pulse combustors [1-3]. Wang [4] analyzed the pulsatile flow in a porous channel. Schneck and Ostrach [5] investigated analytically the pulsatile blood flow in a channel of small exponential divergence by linear approximation for low mean Reynolds number. Radhakrishnamacharya and Maiti [6] studied the heat transfer characteristics of pulsatile flow in a porous channel by using perturbation technique. Malathy and Srinivas [7] made an analytical investigation on pulsatile hydromagnetic flow between two permeable beds. Shawky [8] studied the pulsatile flow with heat transfer of dusty magnetohydrodynamic Ree-Eyring fluid through a channel. Shit and Roy [9] investigated the pulsatile flow and heat transfer of a magneto-micropolar fluid through a stenosed artery under the influence of body acceleration.

Studies pertaining to the non-Newtonian fluids are important because of its applications in biological sciences and industry such as flow of blood, food mixing and cyme movement in the intestine, paint, polymer solutions, flow of nuclear fuel slurries and flow liquid metal and alloys [10-15]. Rosca and Pop [14] analysed the flow and heat transfer characteristics of Powell-Eyring fluid over a shrinking surface in a parallel free stream. Beg et al. [15] studied numerically the transient pulsatile magneto-hemodynamic non-Newtonian flow and drug diffusion in a porous medium channel. Nanofluid is a mixture of nano-sized particles suspended in a base fluid, is used to enhance the rate of heat transfer through its higher thermal conductivity compared to the base fluid. Nanofluids play vital role to significantly increase the heat transfer rates in many areas such as industrial cooling applications, nuclear reactors, transportation industry, micro-electromechanical systems, heat exchangers, chemical catalytic reactors, fiber and granular insulation, packed beds, petroleum reservoirs and nuclear waste repositories and biomedical applications([16]-[20]). Hatami et al. [20] have analysed analytically and numerically the flow and heat transfer characteristics of a non-Newtonian third grade nanofluid in porous medium of a hollow vessel in presence of magnetic field. In their simulation the blood is considered as the base third grade non-Newtonian fluid and gold (Au) as
nanoparticles are added to it. Nadeem and Saleem [21] analytically studied the problem of unsteady Eyring Powell non-Newtonian nanofluid on a rotating cone. Ashraf et al. ([22] several references therein) obtained analytical solutions for three-dimensional flow of Eyring-Powell nanofluid by convectively heated exponential stretching sheet using homotopy analysis method. Hayat et al. [23] analysed the effect of thermal radiation on MHD flow of Powell-Eyring nanofluid induced by a stretching cylinder. Islami et al. [24] investigated the flow and heat transfer of a non-Newtonian nanofluid in two-dimensional parallel plate microchannel with and without micromixers. In their investigation the nanofluid was composed of copper oxide (CuO) nanoparticles and Carboxymethyl Cellulose as the non-Newtonian base fluid. Hatami and Ganji [25] studied analytically and numerically the flow and heat transfer of non-Newtonian nanofluid passing through the porous media between two coaxial cylinders. The authors’ considered sodium alginate (SA) as the base non-Newtonian nanofluid and titanium dioxide (TiO\textsubscript{2}) nanoparticles were added to the base fluid. Yadav et al. [26] examined analytically the thermal conduction in a horizontal layer of porous medium saturated with a Kuvshiniski viscoelastic nanofluid. Rana et al. [27] studied numerically the steady, magnetohydrodynamic boundary layer flow of nanofluid past a permeable stretching sheet with thermal radiation and slip effects.

From the literature survey and to the best of authors’ knowledge, the study pertaining to pulsatile flow of blood carrying gold nanoparticles in a porous space has not been explored yet. Such a consideration is of great value in biomedical and engineering research. Gold nanoparticles with small size are very important in biomedical science and they can be used to activate or inhibit the growth of blood vessels [20]. Hence the main objective of the present investigation is to study the pulsating flow of Eyring-Powell nanofluid in a porous space in the presence of thermal radiation. Analytical solutions are obtained for flow variables. The effects of various parameters on velocity, temperature and heat transfer rate are investigated.
2. FORMULATION OF THE PROBLEM

The constitutive equation for an isotropic and incompressible flow of Eyring-Powell nanofluid can be written as [8]

\[
\tau_{ij} = \mu_{nf} \frac{\partial V_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1} \left( \frac{1}{c} \frac{\partial V_i}{\partial x_j} \right),
\]

(2.1)

where \( \tau_{ij} \) is the stress tensor of the fluid, \( V_i \) is the velocity components of the fluid, the constants \( \mu_{nf}, \beta \) and \( c \) are characteristic properties of the fluid. Since \( \sinh^{-1} x \approx x, \ |x| \ll 1 \), then

\[
\tau_{ij} = \mu_{nf} \frac{\partial V_i}{\partial x_j} + \frac{1}{\beta c \mu_{nf}} \frac{\partial V_i}{\partial x_j} = \mu_{nf} (1 + \frac{1}{\beta c \mu_{nf}}) \frac{\partial V_i}{\partial x_j}.
\]

(2.2)

Consider the pulsatile flow of Eyring-Powell nanofluid between two infinite long parallel plates embedded in porous medium. As shown in Figure 1, the Cartesian coordinate system is used with the x-axis along the lower plate while the y-axis is normal to it. The lower and upper plates are maintained at uniform temperature \( T_0 \) and \( T_1 \) (\( T_1 > T_0 \)) respectively. Since the plates are infinite extend, all physical quantities excepting pressure may be taken as a function of \( y \) and \( t \) only. Under these assumptions of the governing equations are

\[
\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho_{nf}} \frac{\partial P^*}{\partial x^*} + \nu_{nf} (1 + \frac{1}{\beta c \mu_{nf}}) \frac{\partial^2 u^*}{\partial y^*}^2 - \frac{\mu_{nf}}{\rho_{nf} k} u^*,
\]

(2.3)

\[
\frac{\partial T^*}{\partial t^*} = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T^*}{\partial y^*}^2 - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial y^*} + \frac{\mu_{nf}}{(\rho C_p)_{nf}} (1 + \frac{1}{\beta c \mu_{nf}}) \left( \frac{\partial u^*}{\partial y^*} \right)^2 + \frac{Q_0}{(\rho C_p)_{nf}} (T^* - T_0).
\]

(2.4)

The corresponding boundary conditions are

\[
u^* = 0, T^* = T_0 \text{ at } y^* = 0,
\]

(2.5)

\[
u^* = 0, T^* = T_1 \text{ at } y^* = h.
\]

(2.6)

where \( u^* \) is velocity component in \( x^* \)-direction, \( \rho_{nf} \) is density of nanofluid, \( P^* \) is the pressure, \( \mu_{nf} \) is dynamic viscosity of the nanofluid, \( k \) is permeability of porous medium, \( k \) is the permeability of porous medium, \( (\rho C_p)_{nf} \) is effective
heat capacitance of nanofluid, \( k_{nf} \) thermal conductivity of nanofluid, \( q_r \) is radiative heat flux, \( Q_0 \) is coefficient of heat source/sink, \( T^* \) is the temperature of the nanofluid and \( h \) is the distance between the plates.

The physical properties of nanofluid such as \( \mu_{nf}, \rho_{nf}, (\rho C_p)_{nf} \) and \( k_{nf} \) are given as [17-19]

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \tag{2.7}
\]

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi \rho_s \tag{2.8}
\]

\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{2.9}
\]

\[
\frac{k_{nf}}{k_f} = \frac{ks + 2k_f - 2\phi(k_f - k_s)}{ks + 2k_f + \phi(k_f - k_s)} \tag{2.10}
\]

where \( \rho_f \) is the density of the base fluid, \( \rho_s \) is density of the nanoparticle, \( \mu_f \) is viscosity of the base fluid, \( \phi \) is the nanoparticles volume fraction, \((\rho C_p)_f\), \((\rho C_p)_s\) are the heat capacitance of the base fluid and nanoparticles respectively and \( k_f, k_s \) are thermal conductivities of the base fluid and nanoparticle respectively. By using the Rosseland approximation for radiative heat flux \( q_r \), is simplified as [18],

\[
q_r = -\frac{4\sigma^*}{3\kappa^*} \frac{\partial T^{*4}}{\partial y^*} \tag{2.11}
\]

where \( \sigma^* \) and \( \kappa^* \) is the Stefan-Boltzmann constant and mean absorption coefficient, respectively. If the temperature differences within the fluid are sufficiently small, \( T^{*4} \) may be expressed as linear function of temperature. Taylor series for \( T^{*4} \) about \( T_1 \), after neglecting higher order terms is given by [18]

\[
T^4 \approx 4T_1^3T^* - 3T_1^4 \tag{2.12}
\]
In view of Eqs. (2.11) and (2.12), Eqn. (2.4) becomes

\[
\begin{aligned}
\frac{\partial T^*}{\partial t^*} &= \frac{k_n}{(\rho C_p)_n} \frac{\partial^2 T^*}{\partial y^*^2} + \frac{1}{(\rho C_p)_n} \left[ \frac{16\sigma^*}{3\kappa^*} T_1^3 \frac{\partial^2 T^*}{\partial y^*^2} \right] + \frac{\mu_{nf}(1 + \frac{1}{\beta c\mu_f})}{(\rho C_p)_n} \left( \frac{\partial u^*}{\partial y^*} \right)^2 \\
&+ \frac{Q_0}{(\rho C_p)_n} (T^* - T_0)
\end{aligned}
\]  

(2.13)

The pulsatile flow is assumed to be induced by the pressure gradient of the form [8]

\[
-\frac{1}{\rho_f} \frac{\partial P^*}{\partial x^*} = A(1 + \epsilon e^{i\omega t}). 
\]  

(2.14)

By introducing the following dimensionless variables and parameters,

\[
\begin{aligned}
u &= \frac{u^* \omega}{A}, \quad t = \frac{t^* \omega}{h}, \\
\eta &= \frac{y^*}{h}, \quad \theta = \frac{T^* - T_0}{T_1 - T_0}, \\
P &= \frac{P^*}{A \rho_f h}, \quad k_0 = \frac{1}{\beta c\mu_f}
\end{aligned}
\]

(2.15)

Eqs. (2.14), (2.3) and (2.13) become

\[
\begin{aligned}
-\frac{\partial P}{\partial x} &= 1 + \epsilon e^{i\omega t} \\
\frac{\partial u}{\partial t} &= -\frac{1}{A_1} \frac{\partial P}{\partial x} + \frac{A_2}{A_1} \left( 1 + \frac{k_0}{A_2} \right) \frac{1}{R} \frac{\partial^2 u}{\partial y^2} - \frac{A_2}{A_1} \frac{1}{R} Da u \\
\frac{\partial \theta}{\partial t} &= \left( \frac{A_4}{A_3} + \frac{4 RD}{3 A_3} \right) \frac{1}{RP_{Pr}} \frac{\partial^2 \theta}{\partial y^2} + \frac{A_2}{A_3} \left( 1 + \frac{k_0}{A_2} \right) Ec \left( \frac{\partial u}{\partial y} \right)^2 + \frac{Q}{A_3 R} \theta
\end{aligned}
\]

(2.16)  

(2.17)  

(2.18)

The corresponding boundary conditions are

\[
\begin{aligned}
u &= 0, \quad \theta = 0 \text{ at } y = 0 \\
u &= 0, \quad \theta = 1 \text{ at } y = 1.
\end{aligned}
\]

(2.19)  

(2.20)

where \(A_1 = (1 - \phi) + \phi \frac{\rho_s}{\rho_f}, \quad A_2 = \frac{1}{(1 - \phi)^2\kappa}, \quad A_3 = (1 - \phi) + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}, \quad A_4 = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \quad Pr = \frac{(\rho C_p)\nu_f}{k_f} \) is the Prandtl number, \(Ec = \frac{\omega\pi(C_p)_f(T_1 - T_0)}{k_f k_s} \) is the Eckert number, \(R = \frac{\omega h^2}{nu_f} \) is the frequency parameter, \(k_0 = \frac{1}{\beta c\mu_f} \) is the non-Newtonian fluid parameter, \(Da = \frac{k}{k_f} \) is Darcy number, \(Rd = \frac{4\sigma^* T_1^3}{k_f k_s} \) is the radiation parameter, \(Q = \frac{Q_0 h^2}{(\rho C_p)_f \nu_f} \) is heat source parameter.
3. SOLUTION OF THE PROBLEM

Since the flow is induced by the pressure gradient of the form given in Eq. (2.16), the velocity and temperature can be expressed as follows:

\[ u = u_0(y) + \epsilon u_1(y)e^{it} \]  
(3.1)

\[ \theta = \theta_0(y) + \epsilon \theta_1(y)e^{it} + \epsilon^2 \theta_2(y)e^{2it} \]  
(3.2)

Using Eqs. (2.16), (3.1) and (3.2) in Eqs. (2.17)-(2.18) and comparing the terms of like powers of \( \epsilon \), we get

\[ \frac{A_2}{A_1} \left( 1 + \frac{k_0}{A_2} \right) \frac{1}{R} u_0'' - \frac{A_2}{A_1} \frac{1}{RDa} u_0 = -\frac{1}{A_1} \]  
(3.3)

\[ \frac{A_2}{A_1} \left( 1 + \frac{k_0}{A_2} \right) \frac{1}{R} u_1'' - \frac{A_2}{A_1} \frac{1}{RDa} u_1 - iu_1 = -\frac{1}{A_1} \]  
(3.4)

\[ \left( \frac{A_4}{A_3} + \frac{4 Rd}{3 A_3} \right) \frac{1}{RPr} \theta_0'' + \frac{A_2}{A_3} \left( 1 + \frac{k_0}{A_2} \right) (u_0')^2 + \frac{Q}{A_3 R} \theta_0 = 0 \]  
(3.5)

\[ \left( \frac{A_4}{A_3} + \frac{4 Rd}{3 A_3} \right) \frac{1}{RPr} \theta_1'' + \frac{A_2}{A_3} \left( 1 + \frac{k_0}{A_2} \right) u_0'u_1 + \frac{Q}{A_3 R} \theta_1 - i\theta_1 = 0 \]  
(3.6)

\[ \left( \frac{A_4}{A_3} + \frac{4 Rd}{3 A_3} \right) \frac{1}{RPr} \theta_2'' + \frac{A_2}{A_3} \left( 1 + \frac{k_0}{A_2} \right) (u_1')^2 + \frac{Q}{A_3 R} \theta_2 - 2i\theta_2 = 0 \]  
(3.7)

where primes denote differentiation with respect to \( y \). The corresponding boundary conditions are

\[ u_0 = 0, \ \theta_0 = 0, \ \text{at} \ y = 0 \]  
(3.8)

\[ u_0 = 0, \ \theta_0 = 1, \ \text{at} \ y = 1 \]  
(3.9)

\[ u_1 = 0, \ \theta_1 = 0, \ \theta_2 = 0, \ \text{at} \ y = 0 \]  
(3.10)

\[ u_1 = 0, \ \theta_1 = 0, \ \theta_2 = 0, \ \text{at} \ y = 1 \]  
(3.11)

By solving Eqs. (3.3)-(3.7) with the corresponding boundary conditions (3.8)-(3.11), one obtains

\[ u_0 = B_7 e^{\sqrt{\text{Pr}_5} y} + B_6 e^{-\sqrt{\text{Pr}_5} y} - B_4/B_3 \]  
(3.12)

\[ u_1 = B_9 e^{\sqrt{\text{Pr}_5} y} + B_8 e^{-\sqrt{\text{Pr}_5} y} - B_4/B_5 \]  
(3.13)

\[ \theta_0 = C_{11} \cos \sqrt{d_2} y + C_{12} \sin \sqrt{d_2} y + C_8 e^{2\sqrt{\text{Pr}_5} y} + C_9 e^{-2\sqrt{\text{Pr}_5} y} + C_{10} \]  
(3.14)
Table 1: Some properties of non-Newtonian fluid and nanoparticles [20]

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$(kg/m$^3$)</th>
<th>Specific heat $C_p$(J/kgk)</th>
<th>Thermal conductivity (W/mk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blood</td>
<td>1050</td>
<td>3617</td>
<td>0.52</td>
</tr>
<tr>
<td>Gold</td>
<td>19300</td>
<td>129</td>
<td>318</td>
</tr>
</tbody>
</table>

$\theta_1 = D_6 e^{\sqrt{C_4}y} + D_5 e^{-\sqrt{C_4}y} + D_1 e^{m_1 y} + D_2 e^{m_2 y} + D_3 e^{m_3 y} + D_4 e^{m_4 y}$ \ (3.15)

$\theta_2 = D_{11} e^{\sqrt{C_6}y} + D_{10} e^{-\sqrt{C_6}y} + D_7 e^{2\sqrt{B_5}y} + D_8 e^{-2\sqrt{B_5}y} + D_9$ \ (3.16)

where $B$‘s, $C$‘s, $D$‘s, $m$‘s are constants given in Appendix.

Further, the heat transfer rates in terms of non-dimensional Nusselt number is defined as

$$Nu = \frac{k_{nf}}{k_f} \frac{\partial T'}{\partial y} / (T_1 - T_0) = -A_4 \left[ \frac{\partial \theta}{\partial y} \right]_{y=0,1}.$$ \ (3.17)

4. RESULTS AND DISCUSSION

In this study under investigation, $\theta_s$ and $\theta_t$ represents the steady and unsteady temperatures of nanofluid respectively. In this section, to understand the influence of different parameters on velocity, temperature and heat transfer rate, the graphical results are presented in Figures 2-6 for blood-gold nanofluid for fixed $R = 5$, $Da = 0.6$, $k_0 = 5$, $Pr = 21$, $Rd = 3$, $Ec = 1$, $Q = 2$, $t = \pi/4$ unless otherwise stated. The thermophysical properties of the base fluid and the nanoparticles are given in Table 1.

The influence of non-Newtonian fluid parameter $k_0$, frequency parameter $R$ and nanoparticles volume fraction $\phi$ on velocity distribution for blood-gold nanofluid is shown in Figure 2. Figure 2a elucidates the effect of $k_0$ on velocity distribution. It can be noticed that the velocity of the nanofluid decreases with an increase in $k_0$ this is due to the fact that the introduction of stretchable stress due to viscoelasticity of blood causes transverse contraction in the boundary layer and hence velocity decreases. Figure 2b reveals the variation of
velocity of nanofluid for different values of $R$. It is observed that the velocity is increasing function of frequency parameter. The effect of nanoparticles volume fraction $\phi$ on velocity distribution is presented in Figure 2c. From this figure one can conclude that the velocity decreases with increasing the nanoparticles volume fraction. Further, for Newtonian case (i.e. $k_0 = 0$ see Figure 2ci) as $\phi$ increases from 0% to 4% there is approximately 10% decrease in velocity while there is approximately 5% decrease in the velocity for non-Newtonian case (i.e. $k_0 = 3$ see Figure 2cii).

![Figure 2: Velocity distribution (a) effect of $k_0$ (b) effect of $R$ (c) effect of $\phi$](image)

The effects of Eckert number $Ec$, non-Newtonian fluid parameter $k_0$, nanoparticles volume fraction $\phi$, radiation parameter $Rd$, heat source parameter $Q$, frequency parameter $R$ and time $t$ on steady and unsteady temperatures of blood-gold nanofluid are shown in Figures 3-5. Figure 3a depicts the effect of $Ec$ on $\theta_s$ and $\theta_t$. It is noticed that both the steady and unsteady temperatures increase with an increase in $Ec$. This increase in temperature may be due to
heat created by viscous dissipation. The unsteady temperature exhibits oscillating character and the maximum is shifted to the boundary layers near the walls (see Figure 3a(ii)). The amplitude of unsteady temperature increases with an increase in $Ec$. The effects of non-Newtonian fluid parameter $k_0$ nanoparticle volume fraction $\phi$ on $\theta_s$ and $\theta_t$ are shown in Figures 3b-3c. One can observe that both the steady and unsteady temperatures of nanofluid decreases with increasing $k_0$ and $\phi$. The amplitude of unsteady temperature decreases with increasing $k_0$ and the maximum is shifted to boundary layers near the walls (see Figure 3(bii)).

Figure 3: Temperature distribution (a) effect of $Ec$ (b) effect of $k_0$ (c) effect of $\phi$
Figure 4a elucidates the effect of radiation parameter $R_d$ on $\theta_s$ and $\theta_t$. It is found that $\theta_s$ and $\theta_t$ are decreasing functions of $R_d$. Figure 4b demonstrates the effect of heat source parameter $Q$ on the temperature of the nanofluid. The temperature increases for a given increase in $Q$. Figure 4c illustrates the variation of the temperature of nanofluid for different values of frequency parameter $R$. It is observed that the temperature of nanofluid increases with an increase in $R$(as noticed in [8]). From Figure 4(cii) it is noticed that for small values of $R$ the unsteady temperature profiles are almost parabolic but oscillates more for large values of $R$ (as observed in [6]). Further, the amplitude of unsteady temperature increases with increasing $R$ and the maximum is shifted to the boundary layers near the walls. Figure 5 shows the effect of $t$ on the unsteady temperature. One can notice that the unsteady temperature profile exhibits oscillating character.

The variations of Nusselt number $Nu$ for different values of $Q$, $R$ and $\phi$ against $t$ are shown in Figure 6. From Figures 6a and 6b it is observed that the heat transfer rate at the wall $y = 1$ increases with increasing $Q$ and $R$ while it decreases at the wall $y = 0$. The amplitude of $Nu$ increases with increasing $Q$ and $R$. From Figure 6c it is clear that the heat transfer rate increases for a given increase in nanoparticles volume fraction $\phi$ at the wall $y = 0$ (see Figure 6(ci)) while it decreases at the wall $y = 1$ (see Figure 6(cii)). Further, $Nu$ exhibits oscillating character with increasing $t$ and $\phi$.

5. CONCLUSION

In this study, considering blood as non-Newtonian fluid, the pulsatile flow of blood carrying gold nanoparticles of Eyring-Powell nanofluid in a porous space in presence of thermal radiation has been investigated. The analytical solutions are obtained for flow variables. The considered problem is important in biomedical field due to its applications in drug delivery system. The gold nanoparticles are efficient drug-carrying and drug-delivery vehicles because they can encapsulate large quantities of therapeutic molecules [20]. The results shows that the velocity of nanofluid decreases with an increase in non-Newtonian fluid parameter and nanoparticles volume fraction while it increases for a given increase in frequency parameter. The results reveals
that the steady and unsteady temperatures of nanofluid increase for a given increase in Eckert number, frequency parameter and heat source parameter while it decreases with increasing non-Newtonian fluid parameter, nanoparticles volume fraction and radiation parameter. Further, the heat transfer rate enhances with increasing frequency parameter and heat source parameter at the upper wall while it decreases at the lower wall. But this behavior is reversed with the variation of nanoparticles volume fraction.
Figure 5: Effect of $t$ temperature on distribution

**APPENDIX**

\[ B_1 = \frac{A_2}{A_1 R} \left( 1 + \frac{k_0}{A_2} \right), \]
\[ B_3 = \frac{B_2}{B_1}, \]
\[ B_5 = \frac{B_2 + i}{B_1}, \]
\[ B_7 = \frac{B_4}{B_3} - B_6, \]
\[ C_1 = \left( \frac{A_4}{A_3} + \frac{4Rd}{3A_3} \right) \frac{1}{R Pr}, \]
\[ C_3 = -\frac{C_2}{C_1}, \]
\[ C_5 = -\frac{2C_2}{C_1}, \]
\[ C_7 = -\frac{C_2}{C_1}, \]
\[ C_9 = \frac{B_6^2 B_3 C_3}{4B_3 + d_2}, \]
\[ C_{11} = -(C_8 + C_9 + C_{10}), \]
\[ B_2 = \frac{A_2}{A_1 R Da}, \]
\[ B_4 = -\frac{1}{A_1 B_1}, \]
\[ B_6 = \frac{B_4}{e^{-\sqrt{B_3}}}, \]
\[ B_8 = \frac{B_4}{e^{-\sqrt{B_5}}}, \]
\[ B_9 = \frac{B_4}{B_3} - B_8; \]
\[ C_2 = \frac{A_2 Ec}{A_3 R} \left( 1 + \frac{k_0}{A_2} \right), \]
\[ C_4 = \frac{i - d_1}{C_1}, \]
\[ C_6 = \frac{2i - d_1}{C_1}, \]
\[ C_8 = \frac{B_7^2 B_3 C_3}{4B_3 + d_2}, \]
\[ C_{10} = \frac{-2B_6 B_7 B_3 C_3}{d_2}. \]
Figure 6: Nusselt number distribution (a) effect of $Q$ (b) effect of $R$ (c) effect of $\phi$.

\[ C_{12} = \left( 1 - C_{11} \cos \sqrt{d_2} + C_8 e^{2\sqrt{B_5}} + C_9 e^{-2\sqrt{B_5}} + C_{10} \right) / \sin \sqrt{d_2}, \]

\[ D_1 = \frac{B_7 B_9 C_5 \sqrt{B_3 B_5}}{m_1^2 - C_4}, \quad D_2 = -\frac{B_7 B_8 C_5 \sqrt{B_3 B_5}}{m_2^2 - C_4}, \]
\[ D_3 = -\frac{B_6 B_9 C_5 \sqrt{B_3 B_5}}{m_3^2 - C_4}, \quad D_4 = -\frac{B_6 B_8 C_5 \sqrt{B_3 B_5}}{m_4^2 - C_4}, \]

\[ D_5 = \left( D_1 \left( e^{\sqrt{C_4}} - e^{m_1} \right) + D_2 \left( e^{\sqrt{C_4}} - e^{m_2} \right) + D_3 \left( e^{\sqrt{C_4}} - e^{m_3} \right) \right) \]
\[ + D_4 \left( e^{\sqrt{C_4}} - e^{m_4} \right) \right) / \left( e^{-\sqrt{C_4}} - e^{\sqrt{C_4}} \right), \]

\[ D_6 = -(D_1 + D_2 + D_3 + D_4 + D_5), \]

\[ D_7 = \frac{B_9^2 B_5 C_7}{4B_5 - C_6}, \quad D_8 = \frac{B_8^2 B_5 C_7}{4B_5 - C_6}, \]
\[ D_9 = \frac{2B_8 B_9 B_5 C_7}{C_6}, \quad d_1 = \frac{Q}{A_3 R^1}. \]
\[ d_2 = \frac{d_1}{C_1}, \quad m_4 = -\left(\sqrt{B_3} + \sqrt{B_5}\right), \]

\[ D_{10} = \left( D_7 \left( e^{\sqrt{C_6}} - e^{2\sqrt{B_5}} \right) + D_8 \left( e^{\sqrt{C_6}} - e^{-2\sqrt{B_5}} \right) + D_9 \left( e^{\sqrt{C_6}} - e^{m_3} \right) + D_4 \left( e^{\sqrt{C_4}} - 1 \right) \right) / \left( e^{-\sqrt{C_6}} - e^{\sqrt{C_6}} \right), \]

\[ D_{11} = -\left( D_7 + D_8 + D_9 + D_{10} \right), \quad m_1 = \sqrt{B_3} + \sqrt{B_5}, \]

\[ m_2 = \sqrt{B_3} - \sqrt{B_5}, \quad m_3 = -\left( \sqrt{B_3} - \sqrt{B_5} \right). \]

**REFERENCES**


